# WORKSHEET 1: TWO EXAMPLES

Let $X$ be an $m \times n$ matrix. Let $I \subseteq \{1, 2, \ldots, m\}$ and $J = \subseteq \{1, 2, \ldots, n\}$ be two subsets of size $k$; we write $\Delta^I_J(M)$ for the $k \times k$ determinant $\det(M_{ij})_{i \in I,\, j \in J}$, where we keep the rows and columns in the same order as in the big matrix. For example,

$$\Delta^{13}_{24}\left(\begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \end{bmatrix}\right) = \det \begin{bmatrix} M_{12} & M_{14} \\ M_{32} & M_{34} \end{bmatrix}.$$

By convention, $\Delta^{\emptyset}_{\emptyset}(M) = 1$.

We say that a matrix $M$ is ***totally nonnegative*** if all of the $\Delta^I_J(M)$ are $\geq 0$.

**Problem 1.1.** Stratify the space of $2 \times 2$ totally nonnegative matrices according to which of the minors $\Delta^I_J$ are zero. How many strata of each dimension (0, 1, 2, 3 and 4) do you obtain?

**Problem 1.2.** Consider the space of $2 \times 2$ totally nonnegative matrices $\left[\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right]$ with $a + b + c + d = 1$; this lives in $\mathbb{R}^3$, so we can draw it. Sketch this space.

**Problem 1.3.** Consider the space of totally nonnegative matrices of the form $\left[\begin{smallmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{smallmatrix}\right]$. Again, stratify this space according to which are zero. How many strata of each dimension do you obtain?

**Problem 1.4.** Consider the space of totally nonnegative matrices of the form $\left[\begin{smallmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{smallmatrix}\right]$ with $x + y + z = 1$. This lives in $\mathbb{R}^2$, so we can draw it. Sketch this space.