Worksheet 4: Products of upper triangular matrices

We write $x_j(t)$ for the matrix which has t in position $(j, j+1)$, which has 1's on the diagonal and 0's everywhere else.

Show that

$$x_i(\mathbb{R}_{>0})x_j(\mathbb{R}_{>0}) = x_j(\mathbb{R}_{>0})x_i(\mathbb{R}_{>0}) \text{ for } |i-j| \geq 2.$$

$$x_i(\mathbb{R}_{>0})x_{i+1}(\mathbb{R}_{>0})x_i(\mathbb{R}_{>0}) = x_{i+1}(\mathbb{R}_{>0})x_i(\mathbb{R}_{>0})x_{i+1}(\mathbb{R}_{>0}).$$

$$x_i(\mathbb{R}_{>0})x_i(\mathbb{R}_{>0}) = x_i(\mathbb{R}_{>0}).$$

Moreover, show that $x_i x_j$ and $x_i x_{i+1} x_i$ are bijections from $\mathbb{R}^2_{>0}$ and $\mathbb{R}^3_{>0}$, respectively, onto their images.