WORKSHEET 8: $N_+ \cap B_- w B_-$ AS A MANIFOLD

First, a warm up problem:

Problem 8.1. Suppose that

$$w = \begin{bmatrix} & 1 & & & \\ & & & 1 & \\ & & & & 1 \\ & & 1 & & \\ 1 & & & & \end{bmatrix}.$$

What is $N_- w \cap w N_+$? What open subset of it is $N_+ B_- \cap (N_- w \cap w N_+)$?

Recall that we would like to know that $N_+ \cap B_- w B_-$ is a manifold of dimension $\ell(w)$.

Problem 8.2. Show that

$$N_+ \cap B_- w B_- \cong (N_+ B_- \cap B_- w B_-)/B_-.$$

Recall that every element of $B_- w B_-$ has a unique factorization in the form $(N_- \cap w N_+ w^{-1}) w B_-$ or, equivalently, $(N_- w \cap w N_+) B_-$.

Problem 8.3. Show that

$$(N_+ B_- \cap B_- w B_-)/B_- \cong N_+ B_- \cap (N_- w \cap w N_+).$$

Problem 8.4. Show that $N_+ B_- \cap (N_- w \cap w N_+)$ is an open subset of $N_- w \cap w N_+$, and $N_- w \cap w N_+ \cong \mathbb{R}^{\ell(w)}$.