PROBLEM SET SIX: DUE FRIDAY, OCTOBER 30

Problem 1. Cancelled, because the main claim was in error.

Problem 2. Let

$$\dot{s}_i = \begin{bmatrix} 1 & & & & & & \\ & 1 & & & & & \\ & & \ddots & & & & \\ & & & 0 & 1 & & \\ & & & -1 & 0 & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{bmatrix}$$

where the 2×2 block is in rows and columns i and $i+1$.

(1) For $w \in S_n$, let $s_{i_1} s_{i_2} \cdots s_{i_N}$ be a reduced word for w. Define $\dot{w}$ to be $\dot{s}_{i_1} \dot{s}_{i_2} \cdots \dot{s}_{i_N}$. Show that $\dot{w}$ is independent of the choice of reduced word.
(2) Show that each nonvanishing right justified minor of $\dot{w}$ is 1.

In class, we showed that, for $g \in N_+ \cap B_- w B_-$, there is a unique $f \in N_- w \cap w N_+$ with $gB_- = fB_-$. The same argument shows that there is a unique $\dot{f} \in N_- \dot{w} \cap \dot{w} N_+$ with $gB_- = \dot{f} B_-$. (You don't have to check this.) This $\dot{f}$ is slightly more useful than f, for the following reason:

(3) Suppose that g is totally nonnegative. Show that each right justified minor of $\dot{f}$ is nonnegative.

Problem 3. What paper/area would you like to learn about for your final presentation?