Reading off the properties of systems of equations from the rref
We have now learned an algorithm to put a matrix into row reduced echelon form.

- Either row is either all 0’s, or else its first nonzero entry is a 1. This 1 is called a *pivot*.

- In a column which contains a pivot, called a *pivot column*, all the other entries are 0.

- The nonzero rows are at the top of the matrix; they are ordered so that the pivots go from left to right as we go down the rows.

\[
\begin{bmatrix}
1 & 2 & 0 & 1 \\
0 & 0 & 1 & 3 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
Here is what that looks like as a system of equations:

\[
\begin{bmatrix}
1 & 2 & 0 & | & 1 \\
0 & 0 & 1 & | & 3 \\
0 & 0 & 0 & | & 0 \\
\end{bmatrix}
\]

\[
x + 2y + z = 1 \\
0 = 3 \\
0 = 0
\]
Reading off properties of the solutions from the rref
Recall that our row reduced equations have the exact same solutions as our additional equations. But, in the row reduced form, it is much easier to see what the solutions are like:

- If we have the equation $0 = [1]$, there are no solutions.
Reading off properties of the solutions from the rref

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- If we have the equation $0 = 1$, there are no solutions.
- Otherwise, there are solutions. If all the variables are pivot variables, then there is a unique solution:

\[
\begin{align*}
c &= 3 \\
r &= 2 \\
s &= 5
\end{align*}
\]
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  \end{align*}$

- In general, we can choose the free variables freely, and express the pivot variables in terms of them.

  $\begin{align*}
  x + 2y &= 1 \\
  z &= 3 \quad \leadsto (x, y, z) = (1 - 2y, y, 3).
  \end{align*}$
In general, we can choose the free variables freely, and express the pivot variables in terms of them.

\[
\begin{bmatrix} x \\ + \\ 2y \\ \hline z \end{bmatrix} = 1 \\
\begin{bmatrix} 0 \end{bmatrix} = 0 \\
\begin{bmatrix} z \end{bmatrix} = 3
\begin{align*}
\leadsto (x, y, z) &= (1 - 2y, y, 3).
\end{align*}
\]

We will often write this in vector notation:

\[
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + y \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}.
\]