We learn by rearranging what we know.	Ludwig Wittgenstein

Vocabulary: Let X be a set. A ***permutation*** of X is a map $f : X \to X$ such that, for every $y \in X$, there is exactly one $x \in X$ with $f(x) = y$. The ***identity*** is the permutation $\mathrm{id}(x) = x$. If f is a permutation of X, the ***inverse*** of f is the permutation f^{-1} so that, if $f(x) = y$ then $f^{-1}(y) = x$. If f and g are two permutations of X, then the ***composition*** of f and g is the permutation $f \circ g$ such that $(f \circ g)(x) = f(g(x))$.

Let G be a set of permutations of X. We say that G is a ***group of permutations of*** X if

(1) The identity permutation is in G.
(2) If f is in G, then f^{-1} is in G.
(3) If f and g are in G then the composition $f \circ g$ is in G.

For $x \in X$, we define the ***stabilizer of*** x, written $\mathrm{Stab}(x)$, to be the set of g in G with $g(x) = x$. We define the ***orbit*** of x, written Gx, to be the set of all elements of the form $g(x)$ as g ranges over G.

Problem 3.1. Show that composition is associative meaning that, for any three permutations f, g and h, we have

$$f \circ (g \circ h) = (f \circ g) \circ h.$$

Problem 3.2. Give an example of a set X and two permutations, f and g, of X such that $f \circ g \neq g \circ f$.

Problem 3.3. Let X be the set of polynomials in the variables r_1, r_2, r_3. Let S_3 be the group of permutations where we permute the variables. For each of the following polynomials, describe its orbit and its stabilizer:

(1) r_1.
(2) $r_1 + \omega r_2 + \omega^2 r_3$, where $\omega = \frac{-1+\sqrt{-3}}{2}$.
(3) $(r_1 + \omega r_2 + \omega^2 r_3)^3$.
(4) $(r_1 - r_2)(r_1 - r_3)(r_2 - r_3)$.

Problem 3.4. Let X be a set and let G be a finite group of permutations of X. For $x \in X$, show that

$$\#(G) = \#\mathrm{Stab}(x) \cdot \#(Gx).$$

Try to write this out carefully!

Problem 3.5. Let X be a set and let G be a group of permutations of X. For $x \in X$ and $g \in G$, show that

$$\mathrm{Stab}(gx) = g\mathrm{Stab}(x)g^{-1}.$$

More notation: We write S_n for the group of all permutations of $\{1, 2, \ldots, n\}$. The group S_n is called ***the symmetric group***.

Alternate notations: The identity permutation can be written 1, e or id. The composition $f \circ g$ may be just written fg.