Math 605

Several Complex Variables Fall term, 2005

Instructor: Prof. Al Taylor
Office: 4858 East Hall
E-mail: taylor@umich.edu

Course description: This is an introductory course in complex function theory in $n > 1$ variables. The subject will be introduced by explaining some striking classical examples that highlight differences with one variable function theory. For example,

- **(i):** the existence of domains on which every analytic function extends to a larger domain;
- **(ii):** the failure of the Riemann mapping theorem, including the existence of bounded domains that are holomorphically equivalent to $\mathbb{C}^2$ and the nonequivalence of the unit ball and the unit polydisk;
- **(iii):** analytic functions cannot have isolated singularities;
- **(iv):** Hartogs' theorem on separate analyticity.

The goal for the rest of the course is to give a solid introduction to many of the basic concepts and theorems of the subject while going deeply into at least one of them. Examples of such topics (too many to include all) are:

- **(a):** plurisubharmonic functions, positive currents, pseudoconvexity, and the geometry of pseudoconvex boundaries;
- **(b):** domains of holomorphy, holomorphic convexity, and the Levi problem;
- **(c):** the $\overline{\partial}-$complex;
- **(d):** zeros of holomorphic functions, algebraic structure of the local ring of holomorphic functions, and the Cousin problems;
- **(e):** integral representations of solutions of the $\overline{\partial}$ equation.
- **(f):** the complex Monge-Ampere operator and pluripotential theory.
- **(g):** the theory of ideals in algebras of holomorphic functions, e.g. Skoda's theorem and/or the description the closed ideals in the algebra of all holomorphic functions on a domain.

Developing any of these topics beyond an introduction leads to deep mathematics. We will pursue one of them far enough to show clearly the methods developed for its study, the particular topic to be selected depending on student interest.

Course grades will be determined from some homework assignments and a final exam or presentation. We will not have an official textbook, but will follow parts of some standard introductions to the field. A reading list is available upon request. Questions can be directed to Al Taylor, either by e-mail or by dropping by my office.