Math 217 Linear Algebra Winter 2011 HW 01

Section 1.1

(20) One Echelon form of this system is

$$\begin{bmatrix} 1 & h & -3 \\ 0 & 4+2h & 0 \end{bmatrix},$$

so the system is always consistent, since the last column is never a pivot column.

(22) One Echelon form for this system is

$$\begin{bmatrix} 1 & -3/2 & h/2 \\ 0 & 0 & 5-3h \end{bmatrix}.$$

Therefore, the system is consistent if and only if $h = 5/3$, which means precisely that the last column is not a pivot column.

(24)
a. True. See the box preceding the subsection titled "Existence and Uniqueness Questions."
b. False. The definition of *row equivalent* requires that there exist a sequence of row operations that transforms one matrix into the other.
c. False. By definition, an inconsistent system has *no* solution.
d. True. This definition of *equivalent systems* is in the second paragraph after equation (2).

(26) Answers may vary. The systems corresponding to the following matrices each have the solution set $x_1 = -2, x_2 = 1, x_3 = 0$. (The tildes represent row equivalence.)

$$\begin{aligned} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} &\sim \begin{bmatrix} 1 & 0 & 0 & -2 \\ 2 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \end{bmatrix} \\ &\sim \begin{bmatrix} 1 & 0 & 0 & -2 \\ 2 & 1 & 0 & -3 \\ 2 & 0 & 1 & -4 \end{bmatrix} \end{aligned}$$

(30) To get from the first to the second, multiply the second row by $-1/2$. To get from the second to the first, multiply the second row by -2.

(33)
$$\begin{array}{rrrcl} 4T_1 & -T_2 & -T_4 & = & 30 \\ -T_1 & +4T_2 & -T_3 & = & 60 \\ & -T_2 & +4T_3 - T_4 & = & 70 \\ -T_1 & & -T_3 + 4T_4 & = & 40 \end{array}$$

(34) $(20, 27.5, 30, 22.5)$

Section 1.2

(10) The matrix has reduced Echelon form

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 0 & 1 & -7 \end{bmatrix},$$

So the general solutions have the form $x_3 = -7$ and $x - 2y = 4$.

(14) The matrix has reduced Echelon form

$$\begin{bmatrix} 1 & 0 & 7 & 0 & 0 & 9 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

so the general form of the solution set is

$$x_5 = 0,$$

$$x_2 - 6x_3 - 3x_4 = 2,$$

$$x_1 + 7x_3 = 9.$$

(20) We begin by row reducing this matrix to

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & h-9 & k-6 \end{bmatrix}.$$

a. This system has no solution when the last column is a pivot column. This happens precisely when $h = 9$ and $k \neq 6$.

b. To have a unique solution means exactly that the second column is a pivot column. Therefore $h \neq 9$.

c. There are infinitely many solutions when the last row is all zeros, i.e., $h = 9$ and $k = 6$.

(24) The system is inconsistent because the pivot in column 5 means that there is a row of the form

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Since the matrix is the *augmented* matrix for a system, the system has no solution.

(26) If each column of the coefficient matrix is a pivot column, then its reduced Echelon form is the 3×3 *identity matrix*

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

so the augmented matrix has the form

$$\begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & a_2 \\ 0 & 0 & 1 & a_3 \end{bmatrix}$$

for some real numbers a_1, a_2, a_3. This means that a_1, a_2, a_3 is the unique solution to the system.

(30) Answers may vary. One system of two equations in three unknowns that is inconsistent is

$$x_1 + 2x_2 + 3x_3 = 4,$$

$$x_1 + 2x_2 + 3x_3 = 5.$$

(31) Yes, a system of linear equations with more equations than unknowns can be consistent. The following system has a solution $x_1 = x_2 = 1$:

$$x_1 + x_2 = 2$$

$$x_1 - x_2 = 0,$$

$$3x_1 + 2x_2 = 5.$$