Book Homework #9 Answers

Math 217 W11

5.2.12. $(-1-\lambda)(4-\lambda)(2-\lambda) = -\lambda^3 + 5\lambda^2 - 2\lambda - 8$

5.2.14. $(-4-\lambda)(7-\lambda)(1-\lambda) = -\lambda^3 + 4\lambda^2 + 25\lambda - 28$

5.2.18. If $h = 6$, $\mathrm{Nul}(A - 5I) = \mathrm{Span}\left\{\begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\3\\1\\0\end{pmatrix}\right\}$. If $h \neq 6$, $\mathrm{Nul}(A - 5I) = \mathrm{Span}\left\{\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\right\}$.

5.2.20. We have a theorem which says that, for any matrix B, the matrices B, B^T have the same determinant. Note that, for any matrix A and any scalar λ, $(A - \lambda I)^T = A^T - \lambda I$, so we have the identity $\det(A^T - \lambda I) = \det(A - \lambda I)$, so A, A^T have the same characteristic polynomial.

5.2.24. Suppose A and B are similar matrices. Then there exists an invertible matrix P such that $B = PAP^{-1}$.

$$\det B = \det(PAP^{-1}) = (\det P)(\det A)(\det P^{-1}) = (\det P)(\det A)(\det P)^{-1} = \det A$$

5.3.6. The eigenvalues are 5 and 4. The 5-eigenspace has a basis $\left\{\begin{pmatrix}-2\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}\right\}$, and the 4-eigenspace has a basis $\left\{\begin{pmatrix}-1\\2\\0\end{pmatrix}\right\}$.

5.3.18. (The answer is not unique.) $P = \begin{pmatrix}-4 & 1 & -2\\3 & 0 & 1\\0 & 3 & 2\end{pmatrix}$; $D = \begin{pmatrix}5 & 0 & 0\\0 & 5 & 0\\0 & 0 & 3\end{pmatrix}$.

5.3.20. (The answer is not unique.) $P = \begin{pmatrix}0 & 2 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{pmatrix}$; $D = \begin{pmatrix}4 & 0 & 0 & 0\\0 & 4 & 0 & 0\\0 & 0 & 2 & 0\\0 & 0 & 0 & 2\end{pmatrix}$.

5.3.24. A is not diagonalizable. Theorem 7b settles the question immediately. Alternatively, suppose that A were diagonalizable. Then A has an eigenbasis $\{\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}\}$. Each of these vectors must be in one of the two eigenspaces, so some two of them are in the same one-dimensional eigenspace, so they are dependent. This contradicts the supposition that the three vectors form a basis.

5.3.28. Suppose that A has n independent eigenvectors. Then these vectors form an eigenbasis of A, so that A is diagonalizable. Thus there exist a diagonal matrix D and an invertible matrix P such that $A = PDP^{-1}$. Then $A^T = (PDP^{-1})^T = (P^T)^{-1}D^TP^T$. Since diagonal matrices are symmetric, we have $A^T = QDQ^{-1}$, where $Q = (P^T)^{-1}$. Thus A^T is diagonalizable, and admit an eigenbasis consisting of n independent eigenvectors.

5.3.32. $\begin{pmatrix}1 & 1\\0 & 0\end{pmatrix}$ is diagonalizable (easy to see because the eigenvalues 1 and 0 are real and distinct) and not invertible (easy to see because of the row of zeroes).