Book Homework #11 Answers

Math 217 W11

5.5.6. $\lambda = 4 - 3i, \begin{pmatrix} i \\ 1 \end{pmatrix}; \lambda = 4 + 3i, \begin{pmatrix} -i \\ 1 \end{pmatrix}$

5.5.12. $\lambda = \pm .3i, \varphi = -\pi/2, r = .3$

5.5.18. (many other answers are possible) $P = \begin{pmatrix} 1 & -3 \\ 2 & 0 \end{pmatrix}; C = \begin{pmatrix} .8 & -.6 \\ .6 & .8 \end{pmatrix}$

5.5.25. Begin by writing $\boldsymbol{v} = (\operatorname{Re} \boldsymbol{v}) + i(\operatorname{Im} \boldsymbol{v})$. Because matrix-vector multiplication is linear,

$$A\boldsymbol{v} = A(\operatorname{Re} \boldsymbol{v}) + A(i \operatorname{Im} \boldsymbol{v}) = A(\operatorname{Re} \boldsymbol{v}) + i(A(\operatorname{Im} \boldsymbol{v})).$$

Since all the entries in A, $\operatorname{Re} \boldsymbol{v}$, and $\operatorname{Im} \boldsymbol{v}$ are real, $A(\operatorname{Re} \boldsymbol{v})$ and $A(\operatorname{Im} \boldsymbol{v})$ are real vectors. Thus we have written $A\boldsymbol{v}$ in the form $\boldsymbol{a} + i\boldsymbol{b}$ for real vectors $\boldsymbol{a}, \boldsymbol{b}$, and such a decomposition is unique: $\operatorname{Re}(A\boldsymbol{v}) = A(\operatorname{Re} \boldsymbol{v})$ and $\operatorname{Im}(A\boldsymbol{v}) = A(\operatorname{Im} \boldsymbol{v})$.

5.5.26.

a) If $\lambda = a - bi$, then

$$A\boldsymbol{v} = \lambda\boldsymbol{v} = (a - bi)(\operatorname{Re} \boldsymbol{v} + i \operatorname{Im} \boldsymbol{v}) = (a \operatorname{Re} \boldsymbol{v} + b \operatorname{Im} \boldsymbol{v}) + i(a \operatorname{Im} \boldsymbol{v} - b \operatorname{Re} \boldsymbol{v}).$$

By Exercise 25, $A(\operatorname{Re} \boldsymbol{v}) = \operatorname{Re}(A\boldsymbol{v}) = a \operatorname{Re} \boldsymbol{v} + b \operatorname{Im} \boldsymbol{v}$, and likewise

$A(\operatorname{Im} \boldsymbol{v}) = \operatorname{Im}(A\boldsymbol{v}) = -b \operatorname{Re} \boldsymbol{v} + a \operatorname{Im} \boldsymbol{v}$.

b) Let $P = [\operatorname{Re} \boldsymbol{v} \quad \operatorname{Im} \boldsymbol{v}]$. By part a, $A(\operatorname{Re} \boldsymbol{v}) = P\begin{pmatrix} a \\ b \end{pmatrix}, A(\operatorname{Im} \boldsymbol{v}) = P\begin{pmatrix} -b \\ a \end{pmatrix}$. Combining these,

$$AP = [A(\operatorname{Re} \boldsymbol{v}) \quad A(\operatorname{Im} \boldsymbol{v})] = \begin{pmatrix} P\begin{pmatrix} a \\ b \end{pmatrix} & P\begin{pmatrix} -b \\ a \end{pmatrix} \end{pmatrix} = P\begin{pmatrix} a & -b \\ b & a \end{pmatrix} = PC$$