Book Homework #12 Answers

Math 217 W11

6.1.20.

a) True. (Example 1 and Theorem 1a.)

b) False. (Would be true if an absolute value was included. See box above Example 2.)

c) True. (definition of orthogonal complement)

d) True. (Pythagorean Theorem)

e) True. (Theorem 3)

6.1.24.

$$\|\boldsymbol{u}+\boldsymbol{v}\|^2 = (\boldsymbol{u}+\boldsymbol{v})\cdot(\boldsymbol{u}+\boldsymbol{v}) = \boldsymbol{u}\cdot\boldsymbol{u}+\boldsymbol{u}\cdot\boldsymbol{v}+\boldsymbol{v}\cdot\boldsymbol{u}+\boldsymbol{v}\cdot\boldsymbol{v} = \|\boldsymbol{u}\|^2+2(\boldsymbol{u}\cdot\boldsymbol{v})+\|\boldsymbol{v}\|^2$$

$$\|\boldsymbol{u}-\boldsymbol{v}\|^2 = (\boldsymbol{u}-\boldsymbol{v})\cdot(\boldsymbol{u}-\boldsymbol{v}) = \boldsymbol{u}\cdot\boldsymbol{u}-\boldsymbol{u}\cdot\boldsymbol{v}-\boldsymbol{v}\cdot\boldsymbol{u}+\boldsymbol{v}\cdot\boldsymbol{v} = \|\boldsymbol{u}\|^2-2(\boldsymbol{u}\cdot\boldsymbol{v})+\|\boldsymbol{v}\|^2$$

Adding these equations gives the desired result.

$$\|\boldsymbol{u}+\boldsymbol{v}\|^2+\|\boldsymbol{u}-\boldsymbol{v}\|^2 = 2\|\boldsymbol{u}\|^2+2\|\boldsymbol{v}\|^2$$

6.1.26. W is the null space of the $1\times n$ matrix $\boldsymbol{u}^T$. Theorem 4.2 applies. In fact W is a plane through the origin ($\boldsymbol{u}$ is a normal vector to the plane, in the calc-3 sense of the word “normal”).

6.1.28. Let $\boldsymbol{w}\in \text{Span}\{\boldsymbol{u},\,\boldsymbol{v}\}$. Then there exist scalars a, b such that $\boldsymbol{w} = a\boldsymbol{u} + b\boldsymbol{v}$. If $\boldsymbol{y}$ is orthogonal to $\boldsymbol{u}$ and to $\boldsymbol{v}$, then $\boldsymbol{y}\cdot\boldsymbol{u} = \boldsymbol{y}\cdot\boldsymbol{v} = 0$. By linearity of the dot product, we have

$$\boldsymbol{y}\cdot\boldsymbol{w} = \boldsymbol{y}\cdot(a\boldsymbol{u}+b\boldsymbol{v}) = a(\boldsymbol{y}\cdot\boldsymbol{u})+b(\boldsymbol{y}\cdot\boldsymbol{v}) = 0,$$

so that $\boldsymbol{w},\boldsymbol{y}$ are orthogonal.