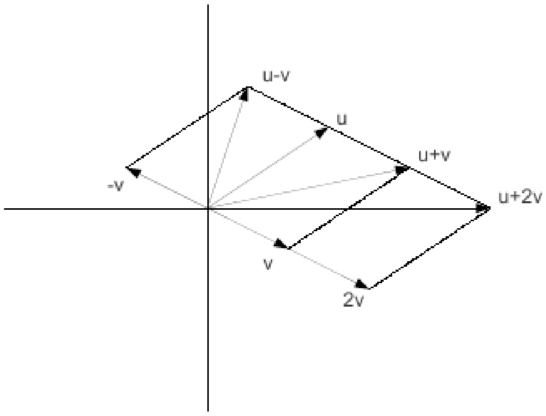
Book Homework #2 Answers

Math 217 W11

1.3.4.



1.3.6.

$$\begin{aligned} -2x_1+8x_2+x_3&=0\\ 3x_1+5x_2-6x_3&=0 \end{aligned}$$

1.3.15. Many, many answers are possible. Here are five possible answers: $0\boldsymbol{v}_1+0\boldsymbol{v}_2=\begin{pmatrix}0\\0\\0\end{pmatrix}$, $2\boldsymbol{v}_1+0\boldsymbol{v}_2=\begin{pmatrix}14\\2\\-12\end{pmatrix}$, $0\boldsymbol{v}_1-1\boldsymbol{v}_2=\begin{pmatrix}5\\-3\\0\end{pmatrix}$, $1\boldsymbol{v}_1+1\boldsymbol{v}_2=\begin{pmatrix}2\\4\\-6\end{pmatrix}$, $2\boldsymbol{v}_1-1\boldsymbol{v}_2=\begin{pmatrix}19\\-1\\-12\end{pmatrix}$.

1.3.18. Suppose $\boldsymbol{y}$ is a linear combination of $\boldsymbol{v}_1,\boldsymbol{v}_2$. Then we have $u\begin{pmatrix}1\\0\\-2\end{pmatrix}+v\begin{pmatrix}-3\\1\\8\end{pmatrix}=\begin{pmatrix}h\\-5\\-3\end{pmatrix}$. This gives three scalar equations.

$$\begin{aligned} u-3v &= h\\ v &= -5\\ -2u+8v &= -3 \end{aligned}$$

The second equation says $v=-5$, and the substitution in the third equation gives $u=-\frac{37}{2}$. Then the first equation will also hold iff $h=-\frac{37}{2}-3(-5)=-\frac{7}{2}$.

The vector $\boldsymbol{y}$ is in the plane spanned by $\boldsymbol{v}_1,\boldsymbol{v}_2$ if and only if $h=-7/2$.

1.3.24.

a) True. (page 31)

b) True. (follows from algebraic properties at the top of page of page 32)

c) False. (page 32, just above Example 4)

d) True. (These are the vectors you get when 0 is the coefficient of $\boldsymbol{v}$.)

e) True. (discussion leading to blue box on page 34)

1.3.32. Many solutions exist. Since none of the three $\boldsymbol{v}_i$ are parallel, any two of them span the whole plane. Then we could take, say, x_3 to be any number and still solve the rearranged equation $x_1\boldsymbol{v}_1 + x_2\boldsymbol{v}_2 = \boldsymbol{b} - x_3\boldsymbol{v}_3$ for x_1, x_2.

1.4.16. This problem can be solved by putting the augmented matrix into echelon form (it is not necessary to reduce). The equation has a solution if and only if $b_1 + 2b_2 + b_3 = 0$. The set of all such $\boldsymbol{b}$ forms a plane through the origin.

1.4.18. There are only three pivot positions in this matrix, but there are four rows. There cannot be a pivot position in every row, so Theorem 4 says that the equation cannot be consistent for *all* $\boldsymbol{y} \in \mathbb{R}^4$.

1.4.20. By Theorem 4, this is essentially the same question as the previous. No.

1.4.24.

a) True. (Thm 3)

b) True. (pages 41–42)

c) True. (Thm 3)

d) True. (Thm 3)

e) False. The system may or may not be consistent, depending on whether the last pivot is or is not in the last column.

f) True. (Thm 3)

1.4.34. Suppose for the sake of contradiction that the columns of A do not span $\mathbb{R}^4$. Then, by Theorem 4, A does not have a pivot position in every row. That is, there are at most three pivots. That is, there is at least one column which does not have a pivot. Thus, whenever we solve an equation $A\boldsymbol{x} = \boldsymbol{b}$, there will be at least one free variable. There can never be a unique solution.

1.4.36. Because matrix-vector multiplication is *linear*, $A(4\boldsymbol{y}) = 4(A\boldsymbol{y}) = 4\boldsymbol{z}$. Thus $4\boldsymbol{y}$ is a solution to the system.

1.5.14. The solution set consists of those vectors of the form $\boldsymbol{x} = \begin{pmatrix} 0 \\ 8 \\ 2 \\ 0 \end{pmatrix} + r\begin{pmatrix} 3 \\ 1 \\ -5 \\ 1 \end{pmatrix}$. This is a line, but not a line through the origin.

1.5.18. The solution set of the homogeneous equation is the plane through the origin spanned by the vectors $\boldsymbol{u} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \boldsymbol{v} = \begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix}$. The solution set to the inhomogenous equation is a parallel plane through $\begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$; i.e., the set of vectors of the form $\begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + r\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix}$ for real numbers r, s.

1.5.24.

a) False. Only one entry needs to be nonzero.

b) True. See Example 2.

c) True. If $\mathbf{0}$ is a solution, then $\boldsymbol{b} = A\mathbf{0} = \mathbf{0}$.

d) True. Example 3 and the following discussion.

e) False. Theorem 6 applies only to consistent systems.

1.5.26.

Solution 1. If $A\boldsymbol{x} = \boldsymbol{b}$ has a solution, then the solution is unique if and only if there are no free variables in the corresponding system of equations. This happens if and only if every column of A is a pivot column. This happens if and only if the equation $A\boldsymbol{x} = \mathbf{0}$ has only the trivial solution.

Solution 2. By Theorem 6, the solution set of $A\boldsymbol{x} = \boldsymbol{b}$ is always either empty or a translation of the solution set of $A\boldsymbol{x} = \mathbf{0}$. In the case where $A\boldsymbol{x} = \boldsymbol{b}$ is consistent, the solution set is a translation of the solution set of $A\boldsymbol{x} = \mathbf{0}$. In particular, the solution set of $A\boldsymbol{x} = \boldsymbol{b}$ consists of a single point iff there is a unique solution to $A\boldsymbol{x} = \mathbf{0}$.

1.5.30.

a) Yes. (The set of solutions form a line.)

b) No.

1.5.32.

a) Yes. (The set of solutions forms a plane.)

b) Yes.

1.5.34. $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$