Book Homework #3 Answers

Math 217 W11

1.7.24. $\begin{pmatrix} \square & \star \\ 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & \square \\ 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

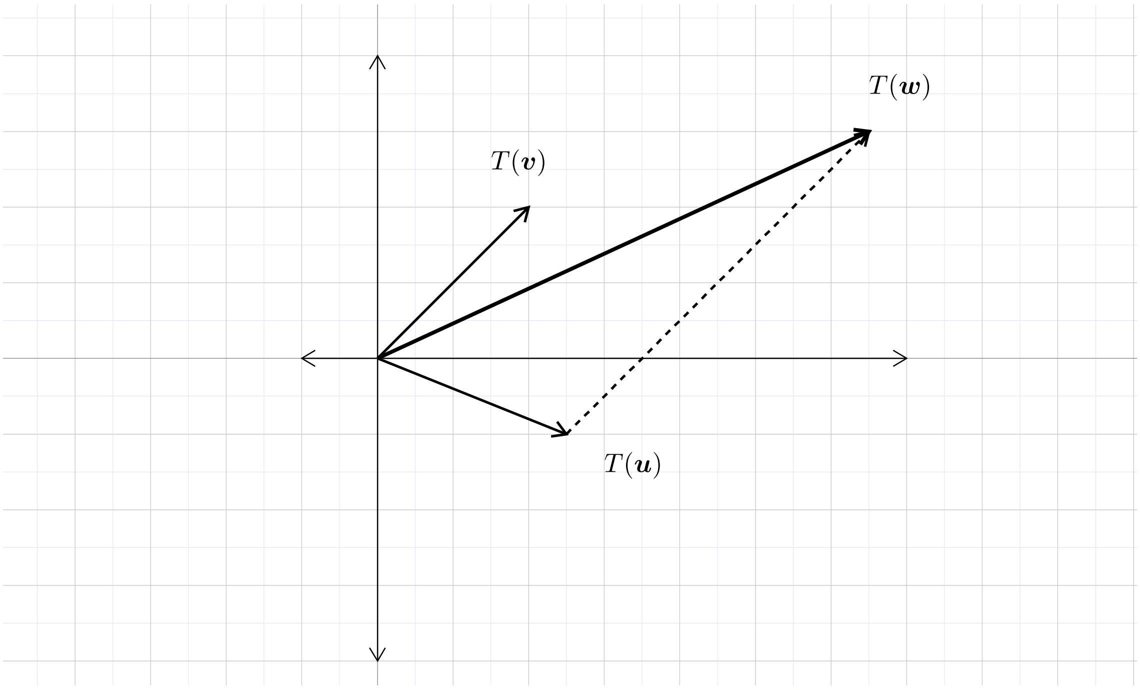
1.7.26. $\begin{pmatrix} \square & \star & \star \\ 0 & \square & \star \\ 0 & 0 & \square \\ 0 & 0 & 0 \end{pmatrix}$

1.7.34. True, by Theorem 9

1.7.36. False. For example, take $\boldsymbol{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \boldsymbol{v}_2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \boldsymbol{v}_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \boldsymbol{v}_4 = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

1.7.38. True. Since $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4\}$ is linearly independent, the equation $x_1\boldsymbol{v}_1 + x_2\boldsymbol{v}_2 + x_3\boldsymbol{v}_3 + x_4\boldsymbol{v}_4 = 0$ has no nontrivial solutions. In particular, there are no solutions in which $x_4 = 0$, so there can be no nontrivial solutions in x_1, x_2, x_3 to the equation $x_1\boldsymbol{v}_1 + x_2\boldsymbol{v}_2 + x_3\boldsymbol{v}_3 + 0\boldsymbol{v}_4 = 0$. That is, there are no nontrivial solutions in x_1, x_2, x_3 to the equation $x_1\boldsymbol{v}_1 + x_2\boldsymbol{v}_2 + x_3\boldsymbol{v}_3 = 0$. By definition, then, $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ is linearly independent.

1.8.18. From the first figure, we can see that (at least approximately) $\boldsymbol{w} = \boldsymbol{u} + 2\boldsymbol{v}$. Thus $T(\boldsymbol{w}) = T(\boldsymbol{u}) + 2T(\boldsymbol{v})$.



1.8.22.

a) True. Paragraph following the definition of a linear transformation.

b) False. The codomain is $\mathbb{R}^m$. Paragraph preceding Example 1.

c) False. This is an existence question. Remark about Example 1(d) following the solution.

d) True. Discussion of the definition of linear transformation.

e) True. Paragraph following eqn (5).

1.8.30. The affine transformation described here maps $\mathbf{0}$ to $\boldsymbol{b}$. A linear transformation maps $\mathbf{0}$ to $\mathbf{0}$. Thus the transformation can be linear only if $\boldsymbol{b}=\mathbf{0}$.

1.8.31. Since $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ is linear dependent, there exist scalars c_1, c_2, c_3 not all zero, such that $c_1\boldsymbol{v}_1 + c_2\boldsymbol{v}_2 + c_3\boldsymbol{v}_3 = \mathbf{0}$. Because T is a linear transformation,

$$c_1T(\boldsymbol{v}_1) + c_2T(\boldsymbol{v}_2) + c_3T(\boldsymbol{v}_3) = T(c_1\boldsymbol{v}_1 + c_2\boldsymbol{v}_2 + c_3\boldsymbol{v}_3) = T(\mathbf{0}) = \mathbf{0}.$$

Because c_1, c_2, c_3 are not all zero, this says that $\{T(\boldsymbol{v}_1), T(\boldsymbol{v}_2), T(\boldsymbol{v}_3)\}$ is linearly dependent.

1.8.32. T does not respect scalar multiplication by negative scalars.

1.8.34. Suppose that $\{\boldsymbol{u}, \boldsymbol{v}\}$ is linearly independent but that $\{T(\boldsymbol{u}), T(\boldsymbol{v})\}$ is linearly dependent. Then there exist scalars c, d not both zero such that $cT(\boldsymbol{u}) + dT(\boldsymbol{v}) = \mathbf{0}$. By linearity of T, we have $T(c\boldsymbol{u} + d\boldsymbol{v}) = \mathbf{0}$. Thus $c\boldsymbol{u} + d\boldsymbol{v}$ is a solution of $T(\boldsymbol{x}) = \mathbf{0}$. Since $\{\boldsymbol{u}, \boldsymbol{v}\}$ is linearly independent, this is not a trivial solution.

1.8.36. Take $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^3$, $c, d \in \mathbb{R}$. Then $c\boldsymbol{u} + d\boldsymbol{v} = (cu_1 + dv_1, cu_2 + dv_2, cu_3 + dv_3)$. Then we can directly compute as follows.

$$\begin{aligned} T(c\boldsymbol{u} + d\boldsymbol{v}) &= T(cu_1 + dv_1, cu_2 + dv_2, cu_3 + dv_3) \\ &= (cu_1 + dv_1, 0, cu_3 + dv_3) \\ &= (cu_1, 0, cu_3) + (dv_1, 0, dv_3) \\ &= c(u_1, 0, u_3) + d(v_1, 0, v_3) \\ &= cT(\boldsymbol{u}) + dT(\boldsymbol{v}) \end{aligned}$$

Thus T is a linear transformation.

1.9.2. $\begin{pmatrix} 1 & 4 & -5 \\ 3 & -7 & 4 \end{pmatrix}$

1.9.4. $\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$

1.9.24.

a) False. Paragraph preceding Example 2.

b) True. Theorem 10.

c) True. Table 1.

d) False. See definition of *one-to-one*.

e) True. Example 5 and its solution.

1.9.26. Use Theorem 12. Onto, but not one-to-one.

1.9.28. Again, Theorem 12. $\boldsymbol{a}_1, \boldsymbol{a}_2$ are independent and span $\mathbb{R}^2$. One-to-one and onto.

1.9.36. For all $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^p$ and for all scalars c, d, we have

$$\begin{aligned} T(S(c\boldsymbol{u} + d\boldsymbol{v})) &= T(cS(\boldsymbol{u}) + dS(\boldsymbol{v})) \\ &= c(T(S(\boldsymbol{u})) + d(T(S(\boldsymbol{v})), \end{aligned}$$

where we are using linearity of S and T, respectively. This shows that $T \circ S$ is linear.