Book Homework #5 Answers

Math 217 W11

3.1.14. Use the fourth row and the last column.

$$\det\begin{pmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{pmatrix} = -3\det\begin{pmatrix} 3 & 2 & 4 & 0 \\ 0 & -4 & 1 & 0 \\ -5 & 6 & 7 & 1 \\ 2 & 3 & 2 & 0 \end{pmatrix}$$
$$= (-3)(-1)\det\begin{pmatrix} 3 & 2 & 4 \\ 0 & -4 & 1 \\ 2 & 3 & 2 \end{pmatrix}$$
$$= (-3)(-1)\left[3\begin{vmatrix} -4 & 1 \\ 3 & 2 \end{vmatrix} + 2\begin{vmatrix} 2 & 4 \\ -4 & 1 \end{vmatrix}\right]$$
$$= 3[-33+36]$$
$$= 9$$

3.1.19. $ad - bc \rightsquigarrow bc - ad$: switching the rows negates the determinant.

3.1.20. $ad - bc \rightsquigarrow kad - kbc$: rescaling the second row by k multiplies the determinant by k.

3.1.21. $-2 \rightsquigarrow -2$: adding a multiple of the first row to the second row does not affect the determinant.

3.1.22. $ad - bc \rightsquigarrow ad - bc$: adding a multiple of the second row to the first row does not affect the determinant.

3.1.23. $-5 \rightsquigarrow -5k$: rescaling the first row by k multiplies the determinant by k.

3.1.24. $2a - 6b + 3c \rightsquigarrow -2a + 6b - 3c$: switching two rows negates the determinant.

3.1.42. The area of the parallelogram is bc (the base is c and the height is b). The determinant of $(\, \mathbf{u} \;\; \mathbf{v} \,)$ is $-bc$, while the determinant of $(\, \mathbf{v} \;\; \mathbf{u} \,)$ is bc.

3.2.34. Since $PP^{-1} = I$, we have $(\det P)(\det P^{-1}) = \det I = 1$. Then, again using Theorem 6, we have $\det(PAP^{-1}) = (\det P)(\det A)(\det P^{-1}) = \det A$.

3.2.36. $\det A^4 = \det O = 0$, but by Theorem 6 $\det A^4 = (\det A)^4$. Thus $\det A = 0$, so A is not invertible.

3.2.40. -2 , 32 , -16 , 1 , -1

3.2.42. $\det(A+B) = \begin{vmatrix} a+1 & b \\ c & d+1 \end{vmatrix} = (a+1)(d+1) - bc = ad + a + d + 1 - bc$. Rearranging this a bit, we obtain $\det(A+B) = (ad - bc) + 1 + (a+d) = \det A + \det B + (a+d)$. Thus, as desired, $\det A + B = \det A + \det B$ iff $a + d = 0$.

3.3.20. The area is given by $\left|\begin{vmatrix} -1 & 4 \\ 3 & -5 \end{vmatrix}\right| = \left|5 - 12\right| = 7$.

3.3.22. The area is given by $\left|\begin{vmatrix} 6 & -3 \\ 1 & 3 \end{vmatrix}\right| = \left|18 - (-3)\right| = 21$.

3.3.30. If a triangle has vertices at $(0,0)$, (x_1, y_1), (x_2, y_2), then its area is half that of the parallelogram with vertices at $(0,0)$, (x_1, y_1), (x_2, y_2), $(x_1 + x_2, y_1 + y_2)$, which is $\det\begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix}$. Thus the area of the triangle is $\left|\frac{1}{2}\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix}\right|$.

Since translation does not affect area, we see that area$[R] =$ area$[R - (x_3, y_3)]$, where $R - (x_3, y_3)$ is the translated copy of R whose vertices are at $(x_1 - x_3, y_1 - y_3)$, $(x_2 - x_3, y_2 - y_3)$, $(0,0)$. Now we compute as follows.

$$\begin{aligned}
\frac{1}{2}\left|\det\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix}\right| &= \frac{1}{2}\left|\det\begin{pmatrix} x_1 - x_3 & y_1 - y_3 & 0 \\ x_2 - x_3 & y_2 - y_3 & 0 \\ x_3 & y_3 & 1 \end{pmatrix}\right| \\
&= \frac{1}{2}\left|\det\begin{pmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_3 & y_2 - y_3 \end{pmatrix}\right| \\
&= \frac{1}{2}\left|\det\begin{pmatrix} x_1 - x_3 & x_2 - x_3 \\ y_1 - y_3 & y_2 - y_3 \end{pmatrix}\right| \\
&= \text{area}(R)
\end{aligned}$$