Book Homework #8 Answers

Math 217 W11

4.5.12. The matrix $\begin{pmatrix} 1 & -3 & -8 & -3 \\ -2 & 4 & 6 & 0 \\ 0 & 1 & 5 & 7 \end{pmatrix}$ row-reduces to $\begin{pmatrix} 1 & 0 & 7 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, so the columns of the matrix span a three-dimensional space (in fact they span $\mathbb{R}^3$).

4.5.14. The nullspace and columnspace of A are each three-dimensional. (There are three columns with pivots and three columns without.)

4.5.22. In the standard coordinates for $\mathbb{P}_3$, the proposed basis polynomials have coordinates $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -4 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 6 \\ -18 \\ 9 \\ -1 \end{pmatrix}$. These coordinate vectors form a invertible triangular matrix, so they are a basis of $\mathbb{R}^4$. The polynomials they represent is a basis of $\mathbb{P}_3$.

4.5.24.

$$\begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & -4 \\ 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 7 \\ -8 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ 3 \end{pmatrix}$$

4.5.26. Let $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$ be a basis of H. Then the basis elements are $n = \dim V$ independent elements of V, so by the Basis Theorem, this basis of H is a basis of V, and $H = V$.

4.6.4. The rank of A is 3, and $\dim \operatorname{Nul} A$ (the nullity) is also 3.

Basis for $\operatorname{Col} A$: $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -1 \\ -3 \\ -2 \end{pmatrix}, \begin{pmatrix} 7 \\ 10 \\ 1 \\ -5 \\ 0 \end{pmatrix} \right\}$.

Basis for $\operatorname{Row} A$: $\{(1, 1, -3, 7, 9, -9), (0, 1, -1, 3, 4, -3), (0, 0, 0, 1, -1, -2)\}$.

Basis for $\operatorname{Nul} A$: $\left\{ \begin{pmatrix} 3 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -9 \\ -4 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 9 \\ 3 \\ 0 \\ 2 \\ 1 \\ 0 \end{pmatrix} \right\}$.

4.6.10. The column space is one-dimensional.

4.6.18.

a) False. The columns of a (row) echelon form of A are not generally in the column space of A.

b) False. Row operations preserve the linear relations among the columns.

c) True. Either use the rank-nullity theorem, or our recipe for constructing our basis of $\operatorname{Nul} A$.

d) True.

e) True. Row operations preserve the row space.

4.6.20. If we represent this system in the form $A\boldsymbol{x} = \boldsymbol{b}$, then A is matrix with 8 columns. Since the solution set has two free variables, Nul A is two-dimensional. By Theorem 14, A has rank 6. Since the columnspace is a 6-dimensional subspace of $\mathbb{R}^6$, Col $A = \mathbb{R}^6$. Thus $A\boldsymbol{x} = \boldsymbol{b}'$ would be consistent for any $\boldsymbol{b}'$.

4.6.22. Write the system in the form $A\boldsymbol{x} = \mathbf{0}$. Then A is a 10×12 matrix. The rank of A is the dimension of a subspace of $\mathbb{R}^{10}$, so the rank is at most 10. By Theorem 14, the dimension of $\operatorname{Nul} A$, which is also the number of free variables, is at least two. Thus the solution space cannot be spanned by a single vector.

4.6.30. $A\boldsymbol{x} = \boldsymbol{b}$ is consistent $\Leftrightarrow$ $\boldsymbol{b} \in \operatorname{Col} A$ $\Leftrightarrow$ $\operatorname{Col} A = \operatorname{Col} [A \quad \boldsymbol{b}]$ $\Leftrightarrow$ $\operatorname{rank} A = \operatorname{rank} [A \quad \boldsymbol{b}]$.