# Book Homework #9 Answers

## Math 217 W11

**4.7.6.**

a) $\begin{pmatrix} 2 & 0 & 0 \\ -1 & 3 & -3 \\ 1 & 1 & 2 \end{pmatrix}$

b) $[\boldsymbol{f}_1 - 2\boldsymbol{f}_2 + 2\boldsymbol{f}_3]_{\mathcal{B}} = \begin{pmatrix} 2 \\ -13 \\ 3 \end{pmatrix}$

**4.7.10.** The change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$ is $\begin{pmatrix} 7 & 2 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} 4 & 5 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 8 & 3 \\ -5 & -2 \end{pmatrix}$. The change-of-coordinates matrix in the other direction is the inverse, $\begin{pmatrix} 2 & 3 \\ -5 & -8 \end{pmatrix}$.

**4.7.12.**

a) True. Basis change matrices are invertible.

b) False. This matrix $P$ changes coordinates in the other direction.

**4.7.14.** The change-of-coordinates matrix is $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ -3 & -5 & 0 \end{pmatrix}$. The inverse of this matrix is $\begin{pmatrix} 10 & -5 & 3 \\ -6 & 3 & -2 \\ 3 & -1 & 1 \end{pmatrix}$, which gives us the following (if we could not see it by inspection).

$$t^2 = 3(1 - 2t + t^2) - 2(2 + t - 5t^2) + 1(1 + 2t)$$

**5.1.16.** $A - 4I = \begin{pmatrix} -1 & 0 & 2 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. The nullspace of the matrix is one-dimensional, spanned by the vector $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$.

**5.1.18.** The eigenvalues are $4, 0, -3$. (Theorem 1.)

**5.1.24.** The $2 \times 2$ identity matrix has only the eigenvalue 1.

**5.1.26.** Suppose that $A^2 = O$, and suppose $\lambda$ is an eigenvalue of $A$. Let $\boldsymbol{v}$ be an eigenvector attached to $\lambda$. Then $A^2\boldsymbol{v} = A(\lambda\boldsymbol{v}) = \lambda(A\boldsymbol{v}) = \lambda^2\boldsymbol{v}$, but $A^2\boldsymbol{v} = O\boldsymbol{v} = \mathbf{0}$. Thus $\lambda = 0$.

**5.1.32.** Typically, 1 is the only real eigenvalue of $T$; the axis of rotation is the 1-eigenspace. If the rotation happens to be a half-turn, then $-1$ is also an eigenvalue; the plane through the origin perpendicular to the axis of rotation is the $(-1)$-eigenspace. (Otherwise, the other eigenvalues are complex.)

**5.4.10.**

a) Let $\boldsymbol{p} = \boldsymbol{p}(t) = a + bt + ct^2 + dt^3$, $\boldsymbol{q} = \boldsymbol{q}(t) = e + ft + gt^2 + ht^3$. Then we compute.

$$T(\boldsymbol{p}) = \begin{pmatrix} a - 3b + 9c - 27d \\ a - b + c - d \\ a + b + c + d \\ a + 3b + 9c + 27d \end{pmatrix} \qquad T(\boldsymbol{q}) = \begin{pmatrix} e - 3f + 9g - 27h \\ e - f + g - h \\ e + f + g + h \\ e + 3f + 9g + 27h \end{pmatrix}$$

$$(r\boldsymbol{p} + s\boldsymbol{q})(t) = (ra + se) + (rb + sf)t + (rc + sg)t^2 + (rd + sh)t^3$$

$$T(r\boldsymbol{p} + s\boldsymbol{q}) = \begin{pmatrix} (ra + se) - 3(rb + sf) + 9(rc + sg) - 27(rd + sh) \\ (ra + se) - (rb + sf) + (rc + sg) - (rd + sh) \\ (ra + se) + (rb + sf) + (rc + sg) + (rd + sh) \\ (ra + se) + 3(rb + sf) + 9(rc + sg) + 27(rd + sh) \end{pmatrix} = rT(\boldsymbol{p}) + sT(\boldsymbol{q})$$

b) The following matrix represents $T$ in the standard coordinates.

$$\begin{pmatrix} 1 & -3 & 9 & -27 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 27 \end{pmatrix}$$

**5.4.12.**

$$\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}^{-1} = \frac{1}{5} \begin{pmatrix} 17 & 9 \\ -16 & -7 \end{pmatrix}$$

**5.4.16.** The characteristic polynomial of $A$ is $\lambda^2 - 5\lambda$, so the eigenvalues are 0 and 5. The 0-eigenspace is spanned by $\left(\begin{smallmatrix} 3 \\ 1 \end{smallmatrix}\right)$; the 5-eigenspace is spanned by $\left(\begin{smallmatrix} 2 \\ -1 \end{smallmatrix}\right)$. The matrix of $T$ relative to the basis $\left\{\left(\begin{smallmatrix} 3 \\ 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 2 \\ -1 \end{smallmatrix}\right)\right\}$ is $\left(\begin{smallmatrix} 0 & 0 \\ 0 & 5 \end{smallmatrix}\right)$.

**5.4.20.** Suppose $A$ is similar to $B$. There exists an invertible matrix $P$ such that $B = PAP^{-1}$. Then $B^2 = (PAP^{-1})(PAP^{-1}) = PA(P^{-1}P)AP^{-1} = PA^2P^{-1}$, so $A^2$ is similar to $B^2$.

**5.4.24.** Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be defined by $T(\boldsymbol{x}) = A\boldsymbol{x}$. Then $A$ represents $T$ in standard coordinates, and $B$ represents $T$ in another coordinate system. Then $\operatorname{rank}(A) = \dim \operatorname{im} T = \operatorname{rank}(B)$.