Proofs Homework Set 4

Due February 2

PROBLEM 4.1. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation. Prove that any three vectors in the image of T are linearly dependent. In other words, show that if $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3 \in \mathbb{R}^2$ then the vectors $T(\mathbf{w}_1), T(\mathbf{w}_2), T(\mathbf{w}_3)$ are linearly dependent.

PROBLEM 4.2. Let A be a $m \times n$ matrix and $\mathbf{b}, \mathbf{c}$ be two vectors in $\mathbb{R}^m$ such that both matrix equations

$$A\mathbf{x} = \mathbf{b} \qquad \text{and} \qquad A\mathbf{y} = \mathbf{c}$$

are consistent. Prove that there exists a vector $\mathbf{d} \in \mathbb{R}^n$ such that the set of solutions $\mathbf{y}$ to the second equation $A\mathbf{y} = \mathbf{c}$ is the set of all vectors of the form $\mathbf{y} = \mathbf{x} + \mathbf{d}$, where $\mathbf{x}$ is any solution of the first equation $A\mathbf{x} = \mathbf{b}$.