Proofs Homework Set 5

MATH 217 — WINTER 2011

Due February 9

PROBLEM 5.1. If A and B are $n \times n$ matrices which are row equivalent, prove that AC and BC are row equivalent for every $n \times n$ matrix C. We will do this in two parts.

(a) Show that if $A \sim B$ (that is, if they are row equivalent), then $EA = B$ for some matrix E which is a product of elementary matrices.

(b) Show that if $EA = B$ for some matrix E which is a product of elementary matrices, then $AC \sim BC$ for every $n \times n$ matrix C.

PROBLEM 5.2. An **upper triangular matrix** is a square matrix in which the entries below the diagonal are all zero, that is, $a_{ij} = 0$ whenever $i > j$. An example is the 4×4 matrix $\begin{bmatrix} 4 & 5 & 10 & 1 \\ 0 & 7 & -1 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$.

Let A be a $n \times n$ upper triangular matrix with nonzero diagonal entries. In this problem, you will build up the pieces necessary to prove that A is invertible and that the inverse of A is also upper triangular.

(a) Prove that the elementary matrices corresponding to the row operations of

- scaling, and
- a replacement move that adds a lower row to a higher row

are upper triangular.

(b) Prove that the two kinds of row operations listed above are sufficient to row-reduce A to the identity matrix. In particular, the matrix A is invertible.

(c) Prove that the product of two upper triangular matrices is upper triangular.

(d) Use these pieces to prove that the inverse of A is upper triangular. (You can get credit for this part even if you didn't do one of the others! Just show that you can put the pieces together to get the desired answer.)