

# Proofs Homework Set 6

MATH 217 — WINTER 2011

*Due February 16*

PROBLEM 6.1.

- (a) If  $A$  is an invertible  $n \times n$  matrix, find  $\det(A^{-1})$  in terms of  $\det(A)$ .
- (b) If  $A$  and  $C$  are  $n \times n$  matrices and  $C$  is invertible, show that  $\det(A) = \det(CAC^{-1})$ .

PROBLEM 6.2. We say that a square matrix  $A$  is **skew-symmetric** if  $A^T = -A$ .

Suppose that  $A$  is a skew-symmetric  $n \times n$  matrix and that  $\mathbf{x}$  is a solution of the homogeneous equation  $(A + I_n)\mathbf{x} = \mathbf{0}$ .

- (a) Show that  $A\mathbf{x} = -\mathbf{x}$ .
- (b) Show that  $\mathbf{x}^T A = \mathbf{x}^T$ .
- (c) Using parts (a) and (b), show that  $\mathbf{x}^T \mathbf{x} = -\mathbf{x}^T \mathbf{x}$ .
- (d) Using part (c), show that  $\mathbf{x} = \mathbf{0}$ .
- (e) Conclude that  $A + I_n$  is invertible.