Proofs Homework Set 7

MATH 217 — WINTER 2011

Due February 23

PROBLEM 7.1. Let V and W be vector spaces, and suppose that $T : V \to W$ is a one-to-one linear transformation. If there are vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ in V such that the vectors $T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_k)$ span W, prove that the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ span V.

PROBLEM 7.2. Let V be a vector space. Suppose that H is a nonempty subset of V such that $\mathrm{Span}\{\mathbf{x}, \mathbf{y}\} \subseteq H$ for all vectors $\mathbf{x}, \mathbf{y} \in H$. Prove that H is a subspace of V.

PROBLEM 7.3. Consider the vector space $C(\mathbb{R})$ of all continuous functions $f : \mathbb{R} \to \mathbb{R}$. Let $Z : C(\mathbb{R}) \to \mathbb{R}$ be defined by $Z(f) = f(0)$.

(a) Prove that Z is a linear transformation.

(b) Prove that Z is onto.

(c) Using part (a), prove that the set $\{f \in C(\mathbb{R}) \mid f(0) = 0\}$ is a subspace of $C(\mathbb{R})$.