# Proofs Homework Set 8

MATH 217 — WINTER 2011

*Due March 9*

PROBLEM 8.1. Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_n\}$ be two bases of a vector space $V$. Prove that the coordinate vectors $\{[\mathbf{b}_1]_{\mathcal{C}}, [\mathbf{b}_2]_{\mathcal{C}}, \ldots, [\mathbf{b}_n]_{\mathcal{C}}\}$ form a basis of $\mathbb{R}^n$.

PROBLEM 8.2. Let $U, V, W$ be three vector spaces and suppose that $T : U \to V$ and $S : V \to W$ are linear isomorphisms (i.e. $T$ and $S$ are one-to-one and onto linear transformations). Prove that their composition $S \circ T$ is also a linear isomorphism. (Recall that the composition $S \circ T$ is the function from $U$ to $W$ defined by $(S \circ T)(\mathbf{x}) = S(T(\mathbf{x}))$ for all $\mathbf{x} \in U$. Don't forget to show that $S \circ T$ is a linear transformation!)

PROBLEM 8.3. Let $V$ be a subspace of $\mathbb{R}^n$ with dimension $n - 1$ and let $\mathbf{x}$ be a vector in $\mathbb{R}^n$ which is not in $V$.

(a) Show that there is a basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n\}$ for $\mathbb{R}^n$ such that $\{\mathbf{b}_1, \ldots, \mathbf{b}_{n-1}\}$ is a basis for $V$ and $\mathbf{b}_n = \mathbf{x}$.

(b) Use part (a) to show that there is a linear transformation $T : \mathbb{R}^n \to \mathbb{R}$ such that $T(\mathbf{x}) = 1$ and the kernel of $T$ is $V$.