Proofs Homework Set 9

MATH 217 — WINTER 2011

Due March 16

PROBLEM 9.1.

(a) Let V be an n-dimensional vector space and let $T : V \to V$ be a linear transformation. Prove that if $\mathrm{Im}(T) = \mathrm{Ker}(T)$, then n is even.

(b) Give an example of such a transformation.

PROBLEM 9.2. Let U and W be subspaces of a finite dimensional vector space V such that $U \cap W = \{\mathbf{0}\}$. Define their sum $U + W := \{u + w \mid u \in U, w \in W\}$, which is also a subspace of V. Let $\mathcal{U}$ be a basis for U and let $\mathcal{W}$ be a basis for W.

(a) Show that $\mathrm{Span}(\mathcal{U} \cup \mathcal{W}) = U + W$.

(b) Show that $\mathcal{U} \cup \mathcal{W}$ is linearly independent.

(c) Conclude from (a) and (b) that $\dim(U + W) = \dim U + \dim W$.