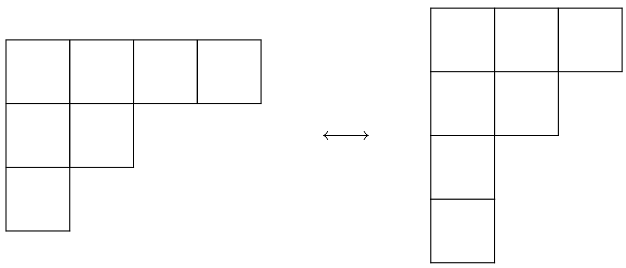
Solution to Problem 2. Suppose P is not graded. Let $[x, y]$ be a minimal interval that is not graded. Then (x, y) is not a nonempty anti-chain, for otherwise $[x, y]$ would be graded of rank 2. Let us suppose (x, y) is connected (and nonempty). Let $z_1, z_2, \ldots, z_r$ be the covers of x. Let $Z_i = (z_i, y)$ be the open intervals between z_i and y.

By our minimality assumption, each $[z_i, y]$ is graded with rank r_i. We need to show that all the r_i are the same. Suppose Z_i intersects Z_j. Then there exists an element $y > w > z_i, z_j$. Let C be a maximal chain from w to y. Let C_i be a maximal chain from z_i to w, and let C_j be a maximal chain from z_j to w. Since the interval $[x, w]$ is graded, C_i and C_j have the same length. Thus $C \cup C_i$ and $C \cup C_j$ have the same length. Thus $r_i = r_j$.

Define an equivalence relation on $(z_1, \ldots, z_r)$ by setting $z_i \sim z_j$ if Z_i and Z_j intersect. The fact that (x, y) is connected implies that all the z's belong to the same equivalence class, so all the r_i are the same.

Idea for Problem 3(b) The idea is to use the *transpose* operation on partitions, acting by swapping rows and columns in Young diagrams. We illustrate this in an example. The partition $(4, 2, 1)$ is transpose to $(3, 2, 1, 1)$.



Then λ' denote the transpose of λ. Then $\lambda \leq \mu$ in dominance order if and only if $\mu' \leq \lambda'$ in dominance order.