

Problem Set 2 Math 637 Winter 2012

You may use any theorems proved in class, or proved in the textbook [Kir] before the corresponding exercise.

- (1) Prove from first principles **one** of the following claims from class.
 - (a) The map $G \times \mathfrak{g} \rightarrow TG$ is an isomorphism of vector bundles. (In particular, you need to carefully show that the map is a diffeomorphism!)
 - (b) The adjoint representation $\text{Ad} : G \rightarrow GL(\mathfrak{g})$ is a homomorphism of Lie groups.
- (2) Show that the exponential map is surjective for the unitary group $U(n)$. (In fact this is true for all compact connected Lie groups.)
- (3) [Kir, Problem 3.1]
- (4) [Kir, Problem 3.3]
- (5) [Kir, Problem 3.5]
- (6) [Kir, Problem 3.7] Kirillov suggests two ways. You must do it the second way. (If you like, you can also do it the first way as well.)
- (7) [Kir, Problems 2.7-2.9] This is essentially one exercise.