Problem Set 4 Math 637 Winter 2012

You may use any theorems proved in class, or proved in the textbook [Kir] before the corresponding exercise.

(1) [Kir, 4.4]
(2) [Kir, 4.5]
(3) [Kir, 4.8]
(4) [Kir, 4.9]
(5) Suppose that all irreducible representations of a compact group G are one-dimensional. Show that G is abelian.
(6) (a) Let G be a compact group. Show that a (strictly decreasing) sequence of closed Lie subgroups $G = G_0 \supsetneq G_1 \supsetneq G_2 \supsetneq \cdots$ eventually terminates.
    (b) A representation $\rho : G \to GL(V)$ is faithful if the kernel of ρ is the identity. Use the Peter-Weyl theorem to show that every compact Lie group G has a faithful representation.
(7) (a) Suppose V is a faithful representation of a compact group G. Verify using the Stone-Weierstrass theorem that the algebra generated by the matrix coefficients of V and of $\bar{V}$ are dense in $C^0(G, \mathbb{C})$. Here $\bar{V}$ denotes the conjugate representation of V, that is ρ is changed to $\bar{\rho}$.
    (b) Let V be a faithful representation of a compact group G. Prove that every irreducible representation of G is a subrepresentation of $V^{\otimes m} \bigotimes (\bar{V})^{\otimes \ell}$ for some $m, \ell \geq 0$.