Comments on Pset 1

(1) For the last question in Problem 1(1), I had expected the following argument: the stated identity is equivalent to the vanishing of some polynomial function on the space of pairs of matrices. The subset consisting of pairs of TNN matrices is Zariski-dense by the following argument: we showed that TNN matrices have the same dimension as the space of matrices, so the Zariski-closure of the subset of TNN matrices is an irreducible component, and hence the whole of, the space of all matrices. Thus the vanishing a polynomial function on pairs of TNN matrices implies that this polynomial vanishes on all pairs of matrices.

However, I'm also happy some of you figured out how to realize any matrix using networks.

(2) For Problem 4(1), have a look at p.108 (Lemma 2) of Fulton's "Young tableaux" for a clean, short proof.

(3) Problem 4(2) was in my opinion the hardest problem. A nice basis is given by the set of products $p_I p_J$ where $I = \{i_1 < i_2 < \cdots < i_k\}$, $J = \{j_1 < j_2 < \cdots < j_k\}$ and $i_\ell \leq j_\ell$ for all $\ell \in [1, k]$. (This set can be identified with the set of semistandard Young tableaux with two columns of length k, filled with numbers in $[n]$.)