Math 463: Introduction to Mathematical Biology

Computer Lab Assignment #3

The Discrete Logistic Equation

The discrete logistic equation,

$$x_{n+1} = f(x_n) = rx_n(1 - x_n)$$

is one of the simplest nonlinear difference equations one can write down. Note that it has just a single parameter, r, and a single quadratic nonlinearity. This equation has been used to describe the growth of populations whose reproductive rate depends on the population size. Despite its simple appearance, this equation has some very interesting and unexpected behavior.

1. Write a simple MATLAB code that computes the iterates of the discrete logistic equation. Use your program to plot, on the same set of axes, a solution that declines to zero together with a solution that rises to a steady state. For this assignment take $x_0 = 0.25$. On your plot, be sure to include the value of the parameter r that was used to generate each curve and compute the steady state value analytically.

2. Now use your program to investigate what happens when r increases from 2.5 to approximately 3.85. On a single sheet of paper, plot the solution to the discrete logistic equation for $r = 2.5, 2.8, 3.2, 3.5, 3.56, 3.83$. Describe what is happening in each graph. Now plot the solution for $r = 3.87$ on a separate piece of paper. Describe the behavior you see.

3. Two point cycles are steady states of the equation

$$x_{n+2} = f(f(x_n)).$$

Now let $f(x) = rx(1 - x)$, with $x \in [0, 1]$. Plot the function $y = f(f(x))$ together with $y = x$ for at least two relevant values of r to show how stable 2-point cycles come into existence. Write down the number of times the curve intersects the line and explain the significance of this.

4. Repeat the above exercise making the appropriate changes to show the existence of 4 and 3 point cycles.