To prove that an integral converges or diverges using the comparison test, your argument must include the following:

1. You must state a correct series of inequalities of functions.

2. You must state the specific valid/relevant interval for which the inequality holds (e.g. for $x \geq 4$...).

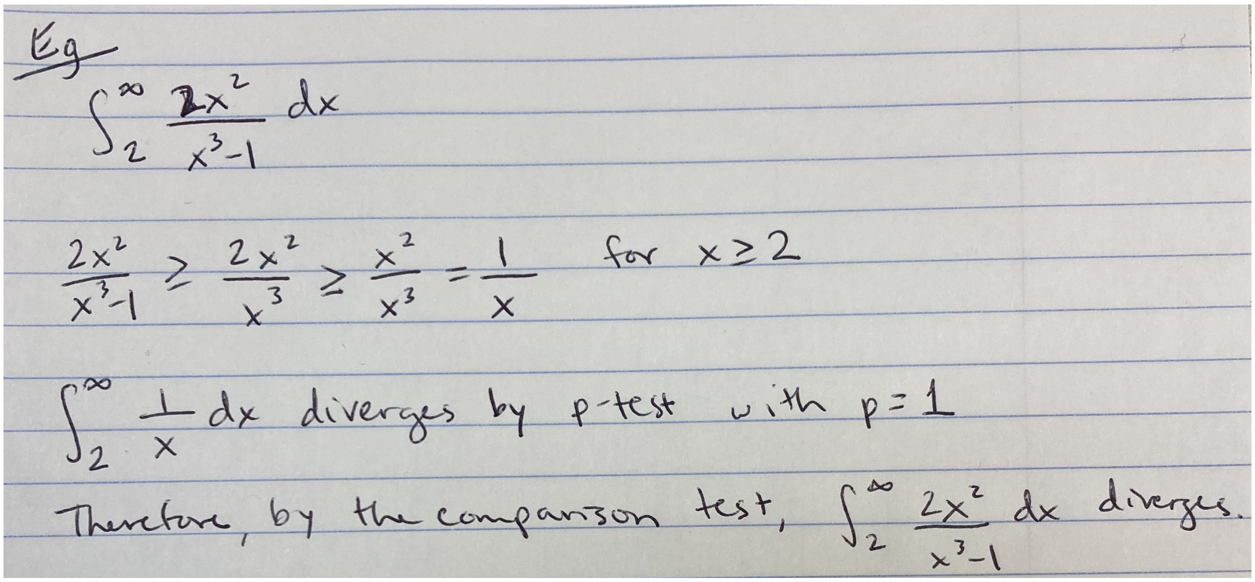
3. You must state that the integral used for comparison either converges or diverges. (Functions don't converge/diverge - integrals do!)

4. Justify how you know the comparison integral converges/diverges either by computing it, using a p-test (must state value of p!), or "exponential decay test".

5. You must conclude that the original interval converges or diverges - don't forget to write what you figured out!

1. Read the expectations for a comparison test. Ask about anything that is unclear.

2. Identify the above items 1–5 in the example below.



Eg

$$\int_2^\infty \frac{2x^2}{x^3-1}\,dx$$

$$\frac{2x^2}{x^3-1} \geq \frac{2x^2}{x^3} \geq \frac{x^2}{x^3} = \frac{1}{x} \quad \text{for } x \geq 2$$

$\int_2^\infty \frac{1}{x}\,dx$ diverges by p-test with $p=1$

Therefore, by the comparison test, $\int_2^\infty \frac{2x^2}{x^3-1}\,dx$ diverges.

3. Write a full argument that the following integral diverges for $r \geq 0$:

$$\int_1^\infty \frac{e^{rx}}{x}dx.$$

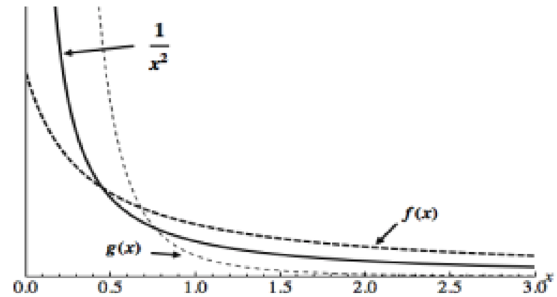
4. [13 points]

a. [8 points] Consider the functions $f(x)$ and $g(x)$ where

$$\frac{1}{x^2} \le g(x) \quad \text{for} \quad 0 < x < \frac{1}{2}.$$

$$g(x) \le \frac{1}{x^2} \quad \text{for} \quad 1 < x$$

$$\frac{1}{x^2} \le f(x) \quad \text{for} \quad 1 < x.$$



Using the information about $f(x)$ and $g(x)$ provided above, determine which of the following integrals is convergent or divergent. Circle your answers. If there is not enough information given to determine the convergence or divergence of the integral circle NI.

i) $\int_1^\infty f(x)dx$ CONVERGENT DIVERGENT **NI**

ii) $\int_1^\infty g(x)dx$ **CONVERGENT** DIVERGENT NI

iii) $\int_0^1 f(x)dx$ **CONVERGENT** DIVERGENT NI

iv) $\int_0^1 g(x)dx$ CONVERGENT **DIVERGENT** NI