# Math 494, Homework 7: due Mar 18

(1) Let $\alpha$ be the unique positive real number such that $\alpha^4 = 7$. Determine which of these extensions are normal: $\mathbb{Q}(i\alpha^2)/\mathbb{Q}$, $\mathbb{Q}(\alpha+i\alpha)/\mathbb{Q}(i\alpha^2)$, $\mathbb{Q}(\alpha + i\alpha)/\mathbb{Q}$.

(2) Give a nice description of the splitting field of each of the following polynomials over $\mathbb{Q}$, and in particular determine the degree of the splitting field (as a field extension of $\mathbb{Q}$): $x^2 - 2$, $x^2 - 1$, $x^3 - 2$, $(x^3 - 2)(x^2 - 2)$, $x^2 + x + 1$, $x^6 + x^3 + 1$ $x^5 - 7$.

(3) For each finite field $L$, determine the sizes of all subfields of $L$ (in terms of the size of $L$). Which extensions of finite fields $L/K$ are normal?

(4) Determine the automorphism group of each finite field (meaning: explicitly name the automorphisms, and also name a well-understood group which is isomorphic to this automorphism group).

(5) Problem 5.2 from chapter 15 of Artin.

(6) Problems 3.1, 3.3, and 4.1 from chapter 16 of Artin.