Math 116 — Practice for Exam 3

Generated November 30, 2017

Name: SOLUTIONS

Instructor: ______________________ Section Number: ________

1. This exam has 2 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2016	2	5	Porcinate	7	
Winter 2012	3	6	fish	9	
Total				16	

Recommended time (based on points): 17 minutes

5. [7 points] The Intern has designed an experiment to stabilize the highly radioactive compound Porcinate. In his experimental setup, the amount $P(t)$ of Porcinate in moles, t hours after the experiment began, should satisfy the differential equation

$$\frac{dP}{dt} - \frac{tP}{\ln(P)} = 0.$$

Use separation of variables to find a solution $P(t)$ satisfying $P(3) = e$.

Solution: We have

$$\int \frac{\ln(P)}{P}\, dP = \int t\, dt,$$

so

$$\frac{(\ln(P))^2}{2} = \frac{t^2}{2} + C$$

for some constant C. The initial condition $P(3) = e$ gives

$$\frac{(\ln(e))^2}{2} = \frac{3^2}{2} + C,$$

so $C = -4$. Hence

$$P(t) = e^{\sqrt{t^2-8}}.$$

6. [5 points] The Intern is also studying a compound called Bovinate. The amount $B(t)$ of Bovinate in moles, t hours after an experiment began, satisfies the differential equation

$$\frac{dB}{dt} = 2B(1-B)(t+B)^2.$$

a. [3 points] List all equilibrium solutions of the differential equation. Indicate whether each is stable or unstable.

Solution: There are two equilibria: $B = 0$ (unstable) and $B = 1$ (stable).

b. [2 points] If the initial amount of Bovinate were 0.5 moles, what would happen to the amount of Bovinate in the long run?

Solution: The amount of Bovinate would approach 1 mole asymptotically from below.

6. [9 points] Let $y(t)$ be the number of fish (**in hundreds**) in an artificial lagoon, where t is measured in years. The function $y(t)$ satisfies the following differential equation

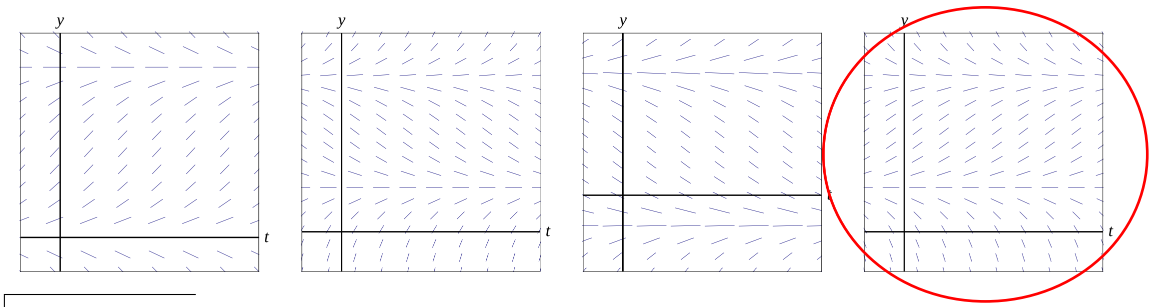
$$\frac{dy}{dt} = y\,(10 - y) - h.$$

where the constant h is the rate at which the fish are harvested from the lagoon.

a. [4 points] Suppose there is no harvesting ($h = 0$). Find the equilibrium solutions of the differential equation. Determine the stability of each equilibrium.

Solution: Equilibrium solutions are found by setting $\frac{dy}{dt} = 0$. So when $h = 0$, we have $y = 0$ and $y = 10$ as equilibrium solutions. Now when $y < 0$ or $y > 10$, we have $\frac{dy}{dt} < 0$. For $0 < y < 10$, we have $\frac{dy}{dt} > 0$. So $y = 0$ is an unstable equilibrium solution and $y = 10$ is a stable equilibrium solutions.

b. [2 points] Suppose the fish are harvested at a rate $h = 9$. Which of the following slope fields may correspond to the differential equation for $y(t)$? Circle your answer.



Solution: The equation is $y' = y(10 - y) - 9$. The equilibrium solutions are $y = 1$ (unstable) and $y = 9$ (stable).

c. [3 points] If (at $t = 0$) there are 200 fish in the lagoon, what is the maximum rate h for harvesting the fish, while still maintaining the fish population in the long run (i.e do not let the fish die out)? (Hint: You do not need to solve the differential equation to answer this question).

Solution: In order for the fish to not die out, we need $\frac{dy}{dt} \geq 0$. That gives us the equation $16 - h \geq 0$, so $h \leq 16$. Therefore 16 is the maximum rate of harvesting for the fish.