Math 116 — Practice for Exam 3

Generated December 4, 2017

Name: ____

Instructor: ______________________ Section Number: ________

1. This exam has 3 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2017	2	3	leaky boat	5	
Winter 2016	2	4	Drake	5	
Fall 2016	2	3	election	5	
Total				15	

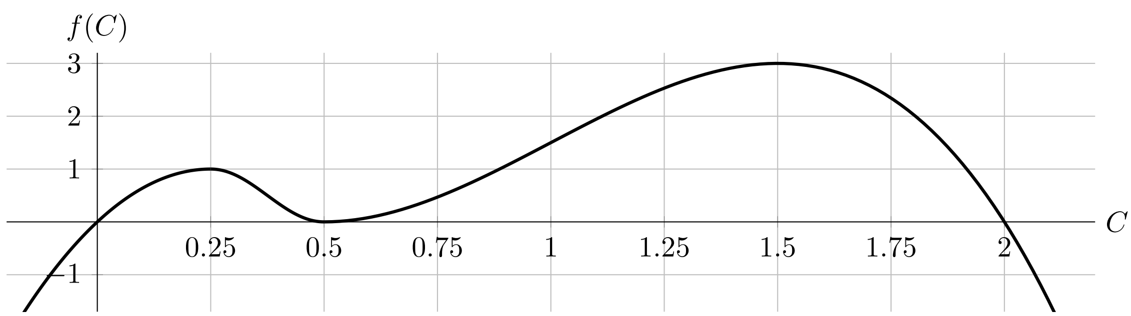
Recommended time (based on points): 14 minutes

3. [5 points] Sasha and her friends are sipping lemonade on her boat when the boat begins to leak through a new hole in the bottom. Water begins to enter the boat at a constant rate of 1.5 gallons per minute. Immediately, they spring into action and begin to scoop the water out of the boat using lemonade pitchers that hold 0.25 gallons of water. That rate that the water is scooped, in scoops per minute, is proportional to the cube root of the volume of water currently in the boat, with constant of proportionality k. Let $W = W(t)$ be the volume of water in the boat, in gallons, t minutes after the leak begins. Write a differential equation that models $W(t)$, and give an appropriate initial condition.

Answer: Differential Equation: ________________________

Initial Condition: ________________________

4. [6 points] Consider the differential equation $\dfrac{dC}{dt} = f(C)$ where $f(C)$ is the function graphed below.



a. [4 points] Identify all equilibrium solutions of this differential equation. Then indicate which of these equilibrium solutions are stable. Write your answers on the answer blanks provided.

Answer: All Equilibrium Solutions: ________________________

Stable Equilibrium Solutions: ________________________

b. [2 points] Suppose that a solution to this differential equation passes through a point with $C = 0.17$. For this solution, what will happen to the value of C as $t \to \infty$?

4. [5 points] Drake is running for president. Suppose $F(t)$ is the **fraction** of the total population of the country who supports him t months after he announces he is running. Drake gains supporters at a steady rate of 2% of the **total population** of the country per month, but he also steadily loses 3% of **his supporters** per month. Write a **differential equation** that models $F(t)$.

5. [6 points] Adele is also running for president. Suppose $P(t)$, the total number of supporters she has in millions t days after she announces, is modeled by the differential equation

$$\frac{dP}{dt} = kP(100 - P)$$

with $k > 0$.

a. [4 points] Find the equilibrium solutions to this differential equation and indicate stabilities for each. Make sure your answer is clear.

b. [2 points] If Adele starts with one million supporters, what is the maximum number of supporters she can get in the long run? You do not need to show your work.

2. [5 points] Find constants A and B so that the function $h(w)$, defined for $w > 0$ by

$$h(w) = Aw^3 + \frac{1}{w},$$

is a solution to the differential equation

$$w^2 \frac{dh}{dw} - 3wh + B = 0$$

satisfying $h(1) = \frac{3}{2}$. Show all your work, and write your final answers in the spaces provided.

$A =$ __________

$B =$ __________

3. [5 points] In a recent presidential election between candidate A and candidate B, Shamcorp's rival company Hawk-I tried fixing the election by changing the votes on some of the ballots. For the last three hours of the election (between 5pm and 8pm), the company gained access to the huge ballot box containing 100 million ballots.

Hawk-I employees removed ballots from the ballot box continuously at a rate of 4 million ballots per hour. Those ballots were removed in proportion to the current ratio in the box. Hawk-I employees then instantly changed the the ballots voting for candidate B to vote for candidate A (leaving any votes for candidate B unchanged) before immediately returning the ballots to the box.

Assume that the ballot box always contains 100 million votes, and that the ballot box only contains votes for candidates A and B.

Write a differential equation that models $a(t)$, the number of ballots voting for candidate A, **in millions**, in the ballot box t hours after Hawk-I began changing votes.

$\dfrac{da}{dt} =$ ________________________