Math 116 — Practice for Exam 2

Generated October 12, 2017

Name: SOLUTIONS

Instructor: ______________________ Section Number: __________

1. This exam has 2 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2010	3	1	rocket	9	
Winter 2011	2	8		15	
Total				24	

Recommended time (based on points): 24 minutes

1. [9 points] When a rocket leaves the gravitational influence of the Earth, it could travel infinitely far away (if we ignore the effects of other celestial bodies). When a rocket of mass m kilograms is h meters above the surface of the Earth, it has a weight of $w = 9.8m\left(\frac{6{,}400{,}000}{6{,}400{,}000+h}\right)^2$ Newtons. Here, $6,400,000$ is the radius of the Earth in meters, and 9.8 is the gravitational constant in $\mathrm{m/s^2}$.

a. [3 points] Approximately how much work is required to lift the rocket Δh additional meters when it is already h meters above the surface of the Earth? Your answer may include m, h, and Δh.

Solution: The weight of the rocket is $9.8m\left(\frac{6{,}400{,}000}{6{,}400{,}000+h}\right)^2$, and so the work needed is

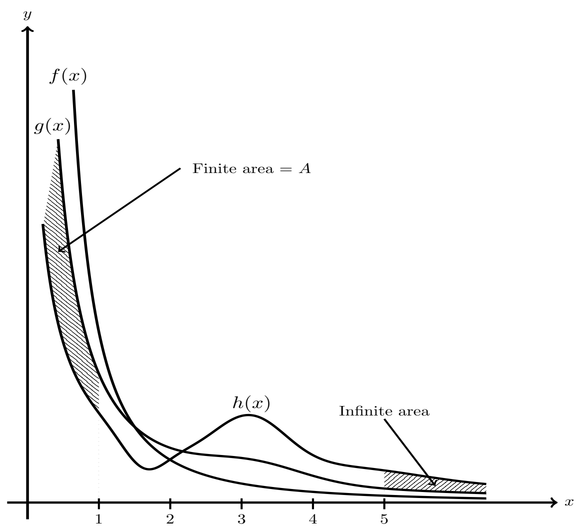
$$9.8m\left(\frac{6,400,000}{6,400,000+h}\right)^2 \Delta h \text{ Joules.}$$

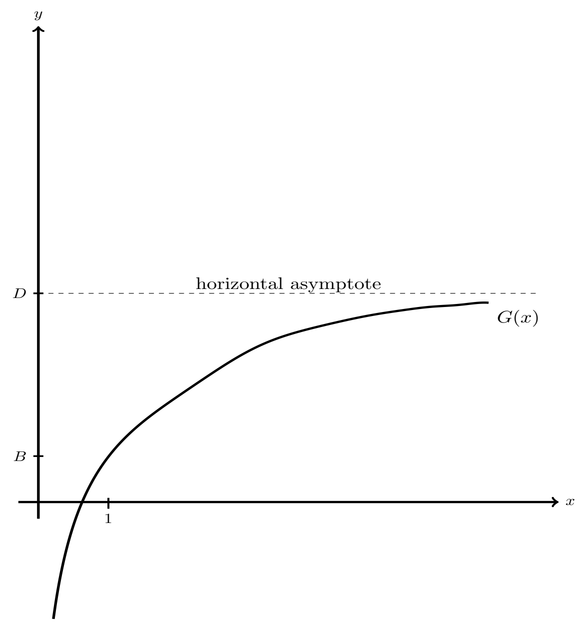
b. [6 points] Figure out the work required to lift the rocket from the surface of the Earth to a height of infinity. Your answer may include m.

Solution: We integrate as h goes between 0 and ∞ :

$$\begin{aligned}
\int_0^\infty 9.8m\left(\frac{6,400,000}{6,400,000+h}\right)^2 dh &= 9.8m \lim_{b\to\infty}\int_0^b \left(\frac{6,400,000}{6,400,000+h}\right)^2 dh \\
&= 9.8m(6,400,000)^2 \lim_{b\to\infty}\int_{6{,}400{,}000}^{b+6{,}400{,}000} \frac{du}{u^2} \\
&= 9.8(6,400,000)^2 m \lim_{b\to\infty}\left[\frac{1}{b+6,400,000} - \frac{1}{6,400,000}\right] \\
&= 9.8(6,400,000)m \text{ Joules.}
\end{aligned}$$

8. [15 points] Graphs of f, g and h are below. Each function is positive, is continuous on $(0, \infty)$, has a horizontal asymptote at $y = 0$ and has a vertical asymptote at $x = 0$. The area between $g(x)$ and $h(x)$ on the interval $(0, 1]$ is a finite number A, and the area between $g(x)$ and $h(x)$ on the interval $[5, \infty)$ is infinite. On the right is a graph of an antiderivative $G(x)$ of $g(x)$. It also has a vertical asymptote at $x = 0$.
Use the information in these graphs to determine whether the following three improper integrals **converge**, **diverge**, or whether there is **insufficient information to tell**. You may assume that f, g and h have no intersection points other than those shown in the graph. **Justify all your answers**.





a. [3 points] $\displaystyle\int_1^\infty h(x)dx$

Solution: Diverges

$$\int_1^\infty h(x)dx = \int_1^5 h(x)dx + \int_5^\infty h(x)dx = \text{finite integral} + \text{divergent integral}$$

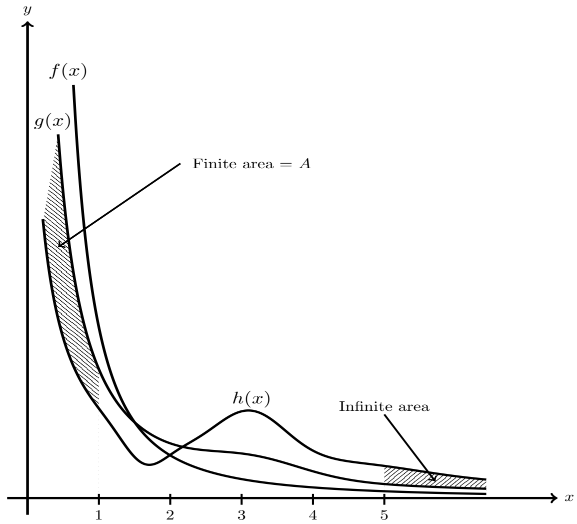
b. [4 points] $\displaystyle\int_0^1 g(x)dx$

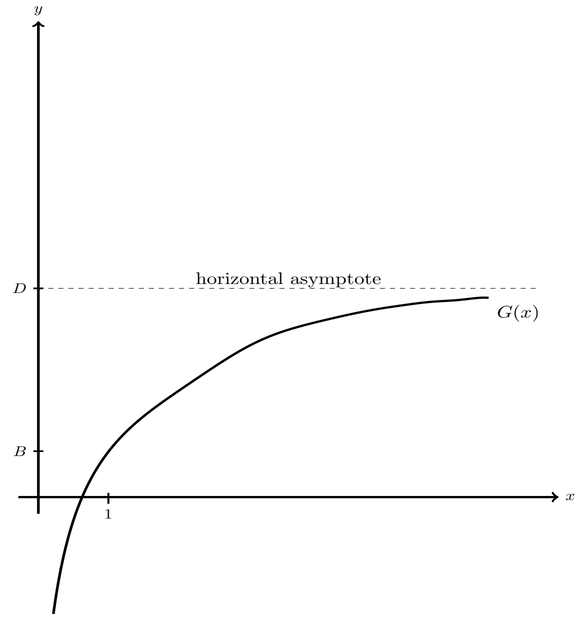
Solution: Diverges

$$\int_0^1 g(x)dx = \lim_{b\to 0^+} \int_b^1 g(x)dx = \lim_{b\to 0^+} G(x)|_b^1 = \lim_{b\to 0^+} G(1) - G(b) = \infty \text{ Diverges}$$

(problem 8 continued)

These graphs are the same as those found on the previous page.





c. [3 points] $\int_0^1 h(x)dx$

Solution: Diverges since

$$\int_0^1 h(x)dx = \int_0^1 g(x)dx - \int_0^1 (g(x) - h(x))dx = \text{ divergent integral+ finite integral}$$

d. [5 points] If $f(x) = 1/x^p$, what are all the possible values of p? **Justify your answer**.

Solution:

$$\begin{aligned}\int_1^\infty g(x)dx &= \lim_{b\to\infty} \int_1^b g(x)dx \\ &= \lim_{b\to\infty} G(b) - G(1) = D - B \text{ converges}\end{aligned}$$

$$\int_3^\infty f(x)dx < \int_3^\infty g(x)dx \text{ convergent integral}$$

Hence $p > 1$.