Math 116 — Practice for Exam 2

Generated October 18, 2017

Name: SOLUTIONS

Instructor: ______________________ Section Number: __________

1. This exam has 2 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2010	2	6		15	
Winter 2010	2	2		10	
Total				25	

Recommended time (based on points): 23 minutes

6. [15 points] Each of the integrals below are improper. Determine the convergence or divergence of each. Make sure you include all the appropriate steps to justify your answers. Approximations with your calculator will not receive credit.

a. [4 points]

$$\int_1^\infty \frac{5-2\sin x}{\sqrt{x^3}}dx$$

Solution: Since $-1 \le \sin x \le 1$ then $3 \le 5 - 2\sin x \le 7$. This yields

$$0 \le \frac{3}{\sqrt{x^3}} \le \frac{5-2\sin x}{\sqrt{x^3}} \le \frac{7}{\sqrt{x^3}}$$

$$0 \le \int_1^\infty \frac{5-2\sin x}{\sqrt{x^3}}dx \le \int_1^\infty \frac{7}{\sqrt{x^3}}dx \qquad \text{converges}$$

b. [5 points]

$$\int_1^2 \frac{x^2}{(x^3-1)^2}dx$$

Solution:

$$\int_1^2 \frac{x^2}{(x^3-1)^2}dx = \lim_{b\to 1^+}\int_b^2 \frac{x^2}{(x^3-1)^2}dx = \lim_{b\to 1^+} -\frac{1}{3(x^3-1)}\Big|_b^2$$

$$= \lim_{b\to 1^+} \frac{1}{3(b^3-1)} - \frac{1}{21} \qquad \text{diverges}$$

c. [6 points]

$$\int_2^\infty \frac{1}{(x^3+7)^{\frac{1}{3}}}dx$$

Solution:

$$\frac{1}{(x^3+7)^{\frac{1}{3}}} \ge \frac{1}{2x} \qquad \text{for } x \ge 1$$

since for $x \ge 1$ we have $x^3 \ge 1$. Multiplying by 7 and adding x^3 we get $8x^3 \ge x^3+7$. Hence $\frac{1}{x^3+7} \ge \frac{1}{8x^3}$ and by taking the cube root on both sides we get $\frac{1}{(x^3+7)^{\frac{1}{3}}} \ge \frac{1}{2x}$. Hence

$$\int_2^\infty \frac{1}{(x^3+7)^{\frac{1}{3}}}dx \ge \int_2^\infty \frac{1}{2x}dx \qquad \text{diverges}$$

2. [10 points] Determine if each of the following integrals diverges or converges. If the integral converges, find the exact answer. If the integral diverges, write "DIVERGES." Show ALL work and use proper notation. Calculator approximations will not receive credit.

a. [5 points] $\int_0^2 \frac{3}{x^{1/3}} dx$

Solution:

$$\begin{aligned} \int_0^2 \frac{3}{x^{1/3}} dx &= \lim_{a\to 0^+} \int_a^2 \frac{3}{x^{1/3}} dx \\ &= \lim_{a\to 0^+} \frac{9}{2} x^{2/3}|_a^2 \\ &= \lim_{a\to 0^+} \left(\frac{9}{2}(2)^{2/3} - \frac{9}{2}a^{2/3}\right) \\ &= \frac{9}{2}(2)^{2/3}. \end{aligned}$$

The integral converges to $\frac{9}{2}(2)^{2/3}$.

b. [5 points] $\int_0^2 \frac{e^{-1/x}}{x^2} dx$

Solution: Make the substitution $u = -\frac{1}{x}$, so $du = \frac{1}{x^2}dx$. Then we have the general antiderivative $\int \frac{e^{-1/x}}{x^2} dx = \int e^u du = e^u + C = e^{-1/x} + C$. That gives us

$$\begin{aligned} \int_0^2 \frac{e^{-1/x}}{x^2} dx &= \lim_{a\to 0^+} \int_a^2 \frac{e^{-1/x}}{x^2} dx \\ &= \lim_{a\to 0^+} e^{-1/x}|_a^2 \\ &= \lim_{a\to 0^+} \left(e^{-1/2} - e^{-1/a}\right) \\ &= e^{-1/2}. \end{aligned}$$

The integral converges to $e^{-1/2}$.