Math 116 — Practice for Exam 2

Generated October 1, 2017

Name: ______________________________

Instructor: ______________________ Section Number: __________

1. This exam has 5 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2014	3	7	chickens vs. robots	14	
Fall 2015	1	8	boxers2	13	
Fall 2016	2	10	tracking chip	14	
Winter 2017	2	6	Venice Beach	13	
Winter 2015	2	1	spinning	16	
Total				70	

Recommended time (based on points): 67 minutes

7. [14 points] Chickens continue to appear around you, and Franklin's army is hesitant to advance.

a. [6 points] Let $F(t)$ give the total number of chickens that have arrived after t seconds. You observe that $F(t)$ obeys the following differential equation

$$\frac{dF}{dt} = e^{-F}t^2.$$

If there are initially 20 chickens, find a formula (in terms of t) for $F(t)$.

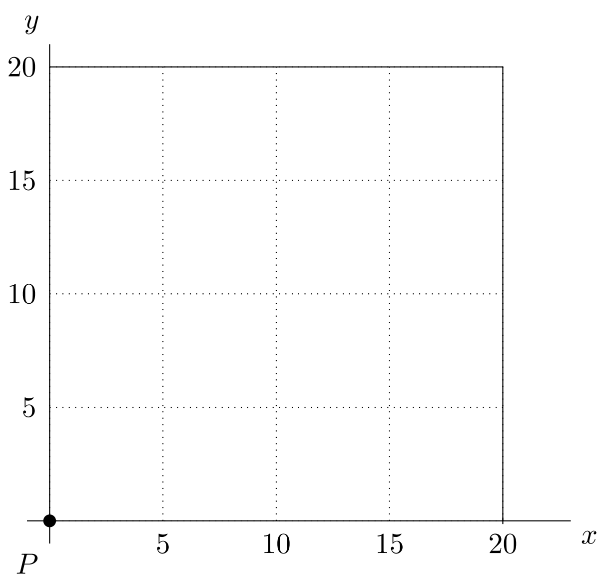
b. [4 points] A large, familiar-looking chicken steps forward from the flock and clucks, "Koo Koo Katcha!". This large chicken waddles towards Franklin following the parametric equations

$$x(t) = \frac{\sin(\pi t) + 1}{\pi} \qquad\qquad y(t) = \ln(t+1)$$

where t is the time, in seconds, after the chicken steps forward from the flock and both x and y are measured in feet. Find the chicken's speed 10 seconds after it steps forward. Include units.

c. [4 points] Franklin says, "`BEEP BOOP BEEP. YOU'RE RIGHT, WHAT HAVE I BECOME?`" A single robot tear falls from Franklin's robot eye. Consider the region in the xy-plane bounded by $y = \dfrac{\sin(x)}{x+2}$, $x = \pi$, $x = 2\pi$, and the x-axis. The volume of Franklin's tear is given by rotating this region around the x-axis. Write an integral giving the volume of Franklin's tear. Do not evaluate this integral.

8. [13 points] During the first round of the rematch between Paul "Stretch" Cassenick and Stephen "Dee" Boxer, Paul's position in the boxing ring t minutes after the 3-minute round began is given by $(x(t), y(t))$ where $x(t)$ and $y(t)$ are Paul's distance from his corner, in feet, in the $x-$ and $y-$directions, respectively. The ring is the 20x20 foot square pictured below, and the point P is Paul's corner. Suppose $x(t) = -8t(t-3)$, and $y(t)$ has values given in the table below and is **linear** between each consecutive pair of t-values in the table.



t	$y(t)$
0	0
1	20
1.5	10
3	0

a. [5 points] On the diagram of the ring, sketch a graph of Paul's path through the ring during the first round of the rematch. Label the points corresponding to Paul's position at $t = 1$ and $t = 1.5$ with their x- and y-coordinates.

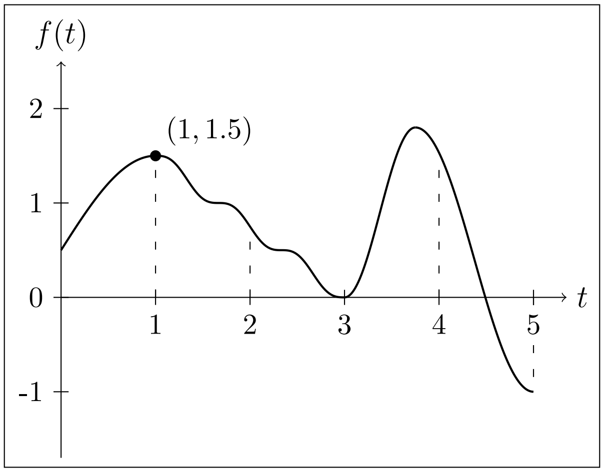
b. [4 points] Find the slope of the tangent line to Paul's path at $t = 2$.

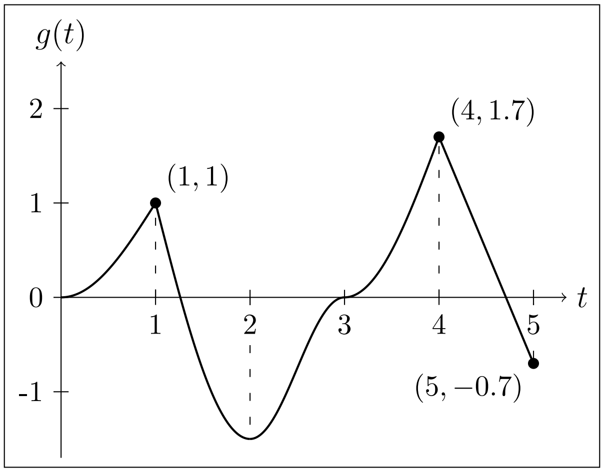
c. [4 points] Write an explicit expression involving integrals that gives the distance Paul traveled during the first minute of the round. Your answer should not contain the letters 'x' or 'y'.

10. [14 points] Fearing that she is losing authority over her robot ward, Dr. Durant has installed a tracking chip in Steph's mainframe. The chip gives Steph's location separately in x- and y-coordinates, where the units of the axes are miles, Dr. Durant's office corresponds to the origin $(x, y) = (0, 0)$, the positive y-axis points north, and the positive x-axis points east. On night 1, Dr. Durant noticed unusual levels of activity; t hours after midnight, Steph began moving according to the parametric equations

$$x = f(t) \qquad\qquad y = g(t),$$

where $f(t)$ and $g(t)$ are plotted below for $0 \leq t \leq 5$.





a. [2 points] When was Steph farthest north and south on night 1? Write your answers in the blanks provided. You do **not** need to show your work.

North: ________ **a.m.** **South:** ________ **a.m.**

b. [3 points] What was Steph's speed at $t = 4.9$ on night 1? You may use the fact that $f'(4.9) = -1$. Include units.

c. [2 points] What direction was Steph moving at $t = 2$ on night 1? Circle only one answer.

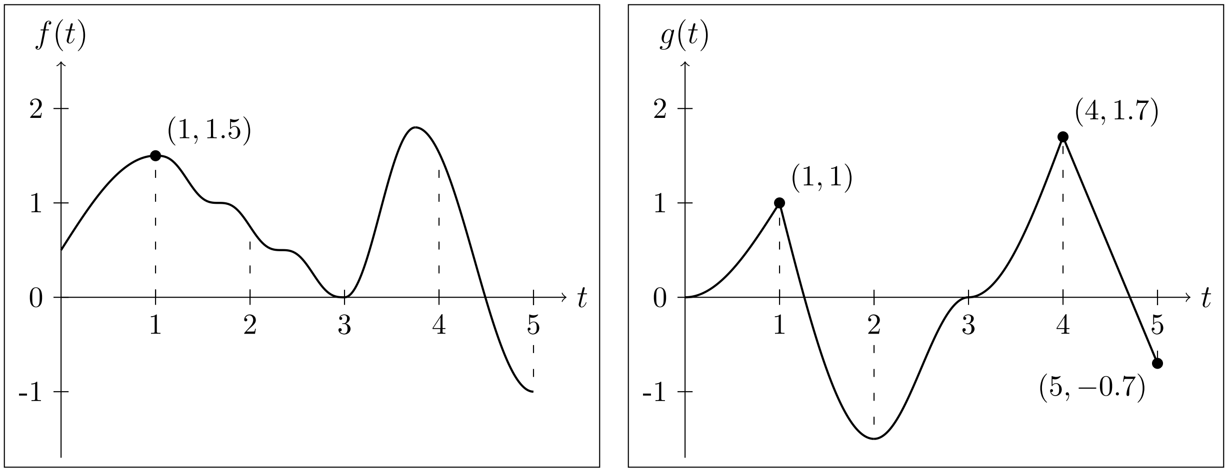
NORTH AND EAST **EAST ONLY** **SOUTH AND EAST**

NORTH AND WEST **WEST ONLY** **SOUTH AND WEST**

10 (continued). Recall that on night 1, Steph's position was given by the parametric equations

$$x = f(t) \qquad\qquad y = g(t),$$

where $f(t)$ and $g(t)$ are plotted below for $0 \le t \le 5$. As before, Dr. Durant's office is at the origin $(x, y) = (0, 0)$, the positive y-axis points north, and the positive x-axis points east.



d. [3 points] How far away was Steph from Dr. Durant's office at $t = 1$ on night 1?

On night 2, Steph's movements were even stranger, following the parametric equations

$$x = \int_0^t f(s)\,ds \qquad\qquad y = \int_0^t g(s)\,ds.$$

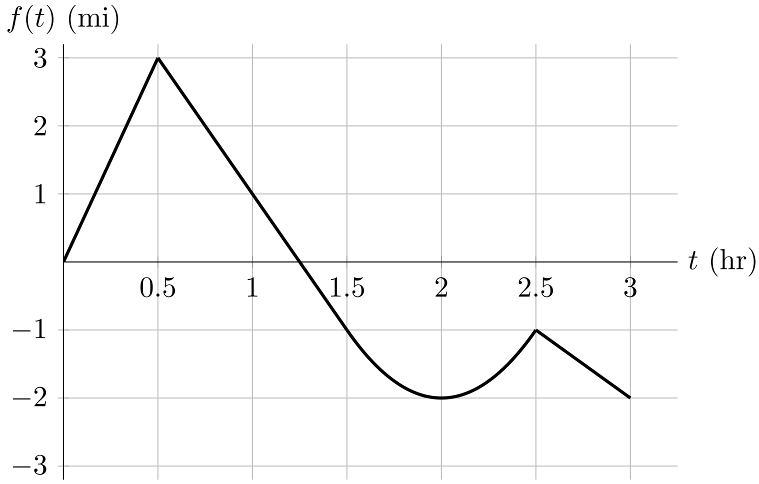
e. [2 points] What direction was Steph moving at $t = 2$ on night 2? Circle only one answer.

NORTH AND EAST **EAST ONLY** **SOUTH AND EAST**

NORTH AND WEST **WEST ONLY** **SOUTH AND WEST**

f. [2 points] Did Steph come to a stop between midnight and 5 a.m. on night 2? If so, at what time(s) did she come to a stop?

6. [13 points] Anderson and Glen decide to take a road trip starting from Venice Beach. They have no worries about getting anywhere quickly, as they enjoy each other's company, so they take a very inefficient route. Suppose that Venice Beach is located at $(0, 0)$ and that Anderson and Glen's position (x, y) (measured in miles) t hours after leaving Venice Beach is given by a pair of parametric equations $x = f(t)$, $y = g(t)$. A graph of $f(t)$ and a formula for $g(t)$ are given below. Note that $f(t)$ is linear on the intervals $[0, 0.5]$, $[0.5, 1.5]$, and $[2.5, 3]$.



$$g(t) = -t^3 + 5t^2 - 3t$$

Note: For each of the following, your final answer should **not** involve the letters f and g.

a. [2 points] If their roadtrip last 3 hours, what are the x– and y– coordinates of their final destination?

b. [3 points] At what speed are they traveling 2 hours into their trip?

c. [4 points] Write, but do not compute, an expression involving one or more integrals that gives the distance they traveled, in miles, in the first half hour of their trip.

d. [4 points] Write down a pair of parametric equations using the parameter s for the line tangent to their path at $t = 2.75$ hours.

Answer: $x(s) =$ ________________ and $y(s) =$ ________________

1. [16 points] Carla and Bobby run a race after spinning in circles for a good amount of time to make themselves dizzy. They start at the origin in the xy-plane and they race to the line $y = 5$. Assume the units of x and y are meters.
Bobby's position in the xy-plane t seconds after the races starts is

$$(-\sqrt{3}t\cos t, \frac{1}{\sqrt{3}}t\sin t)$$

and Carla's position in the xy-plane t seconds after the race starts is

$$(t\sin t, -t\cos t).$$

a. [4 points] Write an integral that gives the distance that Carla travels during the first two seconds of the race. Do not evaluate your integral.

b. [3 points] Find Carla's speed at $t = \pi$.

Carla's speed at $t = \pi$ is ____________________

c. [4 points] Carla and Bobby are so dizzy that they run into each other at least once during the race. Find the first time $t > 0$ that they run into each other, and give the point (x, y) where the collision occurs.

They first run into each other at $t =$ ____________________

The collision occurs at $(x, y) =$ ____________________

d. [5 points] Bobby's phone flies out of his pocket at $t = \pi/2$. It travels in a straight line in the same direction as he was moving at $t = \pi/2$. Find the equation of this line in Cartesian coordinates.

The equation for the line is ____________________