Math 116 — Practice for Exam 2

Generated October 11, 2017

Name: SOLUTIONS

Instructor: ______________________ Section Number: __________

1. This exam has 2 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2014	3	5	wild chickens	11	
Winter 2016	2	2	Leia Han Luke	13	
Total				24	

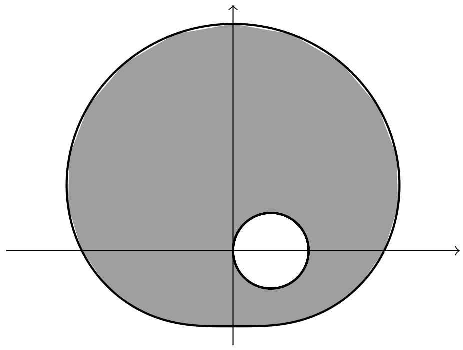
Recommended time (based on points): 25 minutes

5. [11 points] Franklin's robot army is surrounding you!

a. [6 points] Consider the polar curves

$$r = \cos(\theta) \qquad\qquad r = \sin(\theta) + 2$$

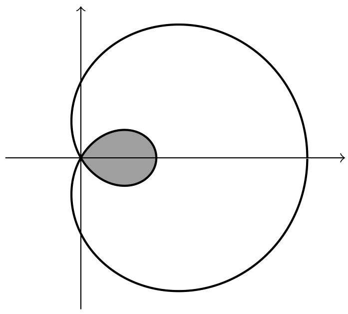
Franklin's robot army occupies the shaded region between these two curves. Write an expression involving integrals that gives the **area** occupied by Franklin's robot army. Do not evaluate any integrals.



Solution:

$$\text{Area} = \frac{1}{2}\int_0^{2\pi} (\sin(\theta) + 2)^2 d\theta - \frac{1}{2}\int_0^{\pi} (\cos(\theta))^2 d\theta$$

b. [5 points] Your friend, Kazilla, pours her magic potion on the ground. Suddenly, a flock of wild chickens surrounds you. The chickens occupy the shaded region enclosed within the polar curve $r = 1 + 2\cos(\theta)$ as shown below. Write an expression involving integrals that gives the **perimeter** of the region occupied by the flock of wild chickens. Do not evaluate any integrals.



Solution: We use the arc length formula:

$$\text{Arc Length} = \int_a^b \sqrt{(r(\theta))^2 + (r'(\theta))^2} d\theta$$

Note that $r'(\theta) = -2\sin(\theta)$. Also, the shaded region of lies between $\theta = 2\pi/3$ and $\theta = 4\pi/3$ (you can see this by setting $r(\theta) = 0$, and testing that $r(\pi) = -1$, so it lies on the boundary of the shaded region.)

$$\text{Arc Length} = \int_{2\pi/3}^{4\pi/3} \sqrt{(1 + 2\cos(\theta))^2 + (-2\sin(\theta))^2} d\theta$$

2. [13 points] Leia and Han are imprisoned in a cell whose door is made out of steel and has a thickness of 3 feet. Luke uses his lightsaber to cut through the door in the shape of the curve given by the polar coordinates equation

$$r = \frac{5}{3 + 2\cos\left(\theta + \frac{\pi}{4}\right)}$$

where r is measured in feet.

a. [6 points] Write an expression involving integrals for the volume of the piece that Luke cuts out of the door.

Solution:

$$3 \cdot \int_0^{2\pi} \frac{1}{2}\left(\frac{5}{3 + 2\cos\left(\theta + \frac{\pi}{4}\right)}\right)^2 d\theta \quad \text{ft}^3$$

b. [7 points] Still considering the polar curve

$$r = \frac{5}{3 + 2\cos\left(\theta + \frac{\pi}{4}\right)}$$

graphed in the xy-plane, write an explicit expression involving integrals for the length of the **part** of the curve that lies **to the right** of the y-axis.

Solution:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\left(\frac{5}{3 + 2\cos\left(\theta + \frac{\pi}{4}\right)}\right)^2 + \left(\frac{10\sin\left(\theta + \frac{\pi}{4}\right)}{\left(3 + 2\cos\left(\theta + \frac{\pi}{4}\right)\right)^2}\right)^2}\, d\theta \quad \text{ft}$$

Alternatively:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta \quad \text{ft}$$

where

$$\frac{dx}{d\theta} = \frac{(-5\sin\theta)(3 + 2\cos(\theta + \pi/4)) + (5\cos\theta)2\sin(\theta + \pi/4)}{[3 + 2\cos(\theta + \pi/4)]^2}$$

and

$$\frac{dy}{d\theta} = \frac{(5\cos\theta)(3 + 2\cos(\theta + \pi/4)) + (5\sin\theta)2\sin(\theta + \pi/4)}{[3 + 2\cos(\theta + \pi/4)]^2}$$