Math 116 — Practice for Exam 3

Generated November 19, 2017

Name: ____

Instructor: ____________________ Section Number: __________

1. This exam has 5 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2012	3	2 c		3	
Fall 2013	2	9	coffee	7	
Fall 2013	2	5 a	pneumonia	2	
Winter 2012	2	9	wire	14	
Fall 2012	2	8	internet cafe	14	
Total				40	

Recommended time (based on points): 56 minutes

2. [13 points] Determine if each of the following sequences is increasing, decreasing or neither, and whether it converges or diverges. Circle all the answers that apply. On parts **a-c**, if the sequence converges, find the limit. No justification is required.

a. [3 points] For $n \geq 1$, let $a_n = 3 + \frac{1}{n}$.

b. [3 points] For $n \geq 1$, let $a_n = (-\frac{\pi}{e})^n$.

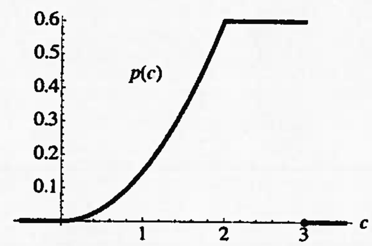
* **c.** [3 points] Let $P(x)$ be the cumulative distribution function of a nonzero probability density function $p(x)$. Define $a_n = P(n)$ for $n \geq 1$.

d. [2 points] For $n \geq 1$, let $a_n = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!}$.

e. [2 points] Let $a_n = \int_2^n \frac{1}{\sqrt{x}-1}\,dx$, for $n \geq 2$.

9. [7 points] Thanks to the Math Department's acquisition of a coffee tank in October, there are now 300 cups of coffee available to the graduate students each day.
The department wants to assess how much of the coffee is drunk and how much is wasted. Let c be the amount of coffee drunk in one day, measured in hundreds of cups of coffee. The probability density function for c is given by

$$p(c) = \begin{cases} \frac{3}{20}c^2 & \text{for } 0 \le c \le 2 \\ \frac{3}{5} & \text{for } 2 \le c \le 3 \\ 0 & \text{otherwise.} \end{cases}$$



a. [4 points] Find the mean of the amount of coffee drunk in one day. Include units. Show all your work.

b. [3 points] Find the median of the amount of coffee drunk in one day. Include units. Show all your work.

5. [10 points] Consider a group of people who have received a new treatment for pneumonia. Let t be the number of days it takes for a person with pneumonia to fully recover. The probability density function giving the distribution of t is

$$f(t) = \frac{10}{(1+at)^2}, \qquad \text{for } t > 0,$$

for some positive constant a.

a. [2 points] Give a practical interpretation of the quantity $\int_3^{10} f(t)dt$. You do not need to compute the integral.

b. [5 points] Find a formula for the cumulative distribution function $F(t)$ of $f(t)$ for $t > 0$. Show all your work. Your answer may include a. Your final answer should not include any integrals.

c. [3 points] Determine the value of a. Show all your work.

9. [14 points] A machine produces copper wire, and occasionally there is a flaw at some point along the wire. The length x of wire produced between two consecutive flaws is a continuous variable with probability density function

$$f(x) = \begin{cases} c(1+x)^{-3} & \text{for } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

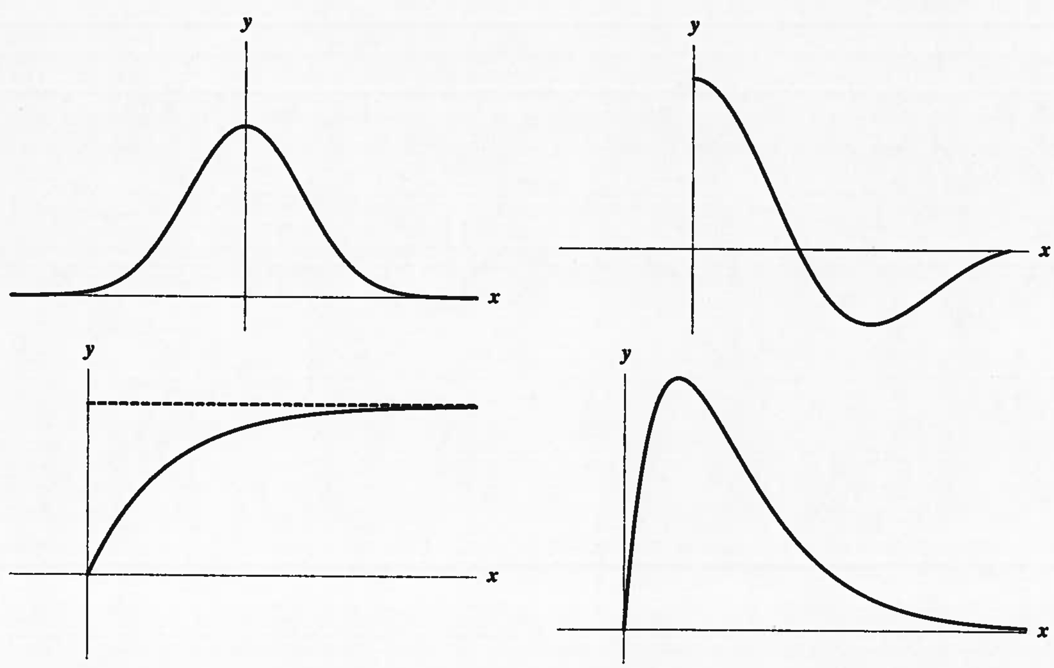
Show all your work in order to receive full credit.

a. [5 points] Find the value of c.

b. [2 points] Find the cumulative distribution function $P(x)$ of the density function $f(x)$. Be sure to indicate the value of $P(x)$ for all values of x.

c. [6 points] Find the mean length of wire between two consecutive flaws.

d. [1 point] A second machine produces the same type of wire, but with a different probability density function (pdf). Which of the following graphs could be the graph of the pdf for the second machine? Circle all your answers.



8. [14 points] A coffee shop offers only one hour of free internet access to all its customers. The time t in hours a customer uses the internet at the coffee shop has a probability density function

$$p(t) = \begin{cases} at\sqrt{1-t^2} & 0 \le t \le 1. \\ 0 & \text{otherwise.} \end{cases}$$

where a is a constant.

a. [4 points] For what value of a is $p(t)$ a probability density function? Find its value without using your calculator.

b. [4 points] Find the cumulative distribution function $P(t)$ of $p(t)$. Make sure to indicate the value of $P(t)$ for all values of $-\infty < t < \infty$. Your final answer should not contain any integrals.

c. [3 points] Find the the probability that a customer is still using the internet after 40 minutes (without using your calculator).

d. [3 points] Find an expression for the mean of this distribution. Use your calculator to compute its value.