MATH 116 — PRACTICE FOR EXAM 2

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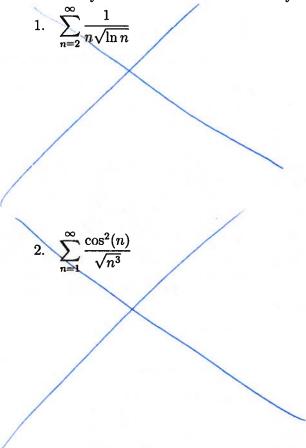
NAME: _____

INSTRUCTOR: _____   SECTION NUMBER: _____

1. This exam has 4 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.

2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.

3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.

4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.

5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.

6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.

7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2012	3	3		12	
Fall 2013	3	2		11	
Fall 2016	3	3		8	
Winter 2013	3	8		8	
Total				39	

Recommended time (based on points): 47 minutes

3. [12 points]

    a. [6 points] State whether each of the following series converges or diverges. Indicate which test you use to decide. Show all of your work to receive full credit.

      1. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

      2. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2(n)}{\sqrt{n^3}}$

    b. [6 points] Decide whether each of the following series converges absolutely, converges conditionally or diverges. Circle your answer. No justification required.

      1. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{n^2+1}}{n^2+n+8}$

      Converges absolutely   Converges conditionally   Diverges

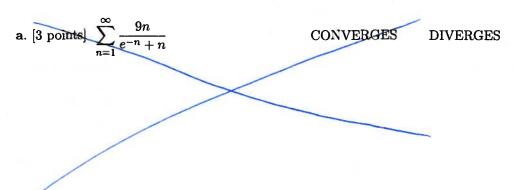   2. $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^{3n}}{5^n}$
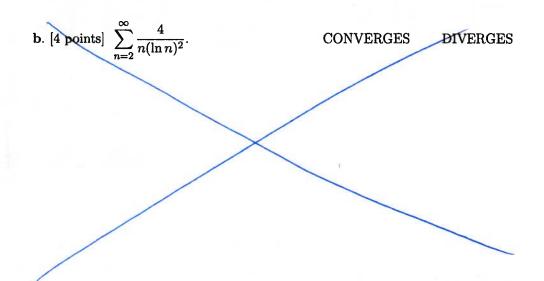
      Converges absolutely   Converges conditionally   Diverges

2. [11 points] Determine the convergence or divergence of the following series. In parts (a) and (b), support your answers by stating and properly justifying any test(s), facts or computations you use to prove convergence or divergence. Circle your final answer. Show all your work.

a. [3 points] $\displaystyle\sum_{n=1}^{\infty} \frac{9n}{e^{-n} + n}$ CONVERGES DIVERGES

b. [4 points] $\displaystyle\sum_{n=2}^{\infty} \frac{4}{n(\ln n)^2}.$ CONVERGES DIVERGES

c. [4 points] Let r be a real number. For which values of r is the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n^2}{n^r + 4}$ absolutely convergent? Conditionally convergent? No justification is required.

Absolutely convergent if : _____

Conditionally convergent if : _____

3. [8 points] For $n = 1, 2, 3, \ldots$ consider the sequence a_n given by

$$a_n = \frac{-1}{2^{(n+1)/2}} \text{ if } n \text{ is odd,} \qquad a_n = \frac{1}{3^{n/2}} \text{ if } n \text{ is even.}$$

a. [2 points] Write out the first 5 terms of the sequence a_n.

b. [2 points] The series $\displaystyle\sum_{n=1}^{\infty} a_n$ is alternating. In a sentence or two, explain why the Alternating Series Test **cannot** be used to determine whether $\displaystyle\sum_{n=1}^{\infty} a_n$ converges or diverges.

c. [4 points] The series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges. Show that it converges, either by using theorems about series, or by computing its exact value.

8. [8 points] Consider the power series

$$\sum_{n=1}^{\infty} \frac{1}{2^n \sqrt{n}} (x-5)^n.$$

In the following questions, you need to support your answers by stating and properly justifying the use of the test(s) or facts you used to prove the convergence or divergence of the series. Show all your work.

a. [2 points] Does the series converge or diverge at $x = 3$?

b. [2 points] What does your answer from part (a) imply about the radius of convergence of the series?

c. [4 points] Find the interval of convergence of the power series.