1. A density function $p(x)$ satisfying (I)-(IV) gives the fraction of years with a given total annual snowfall (in m) for a city.

   (a) $\int_0^{0.5} p(x)dx = 0.1$

   (b) $\int_0^{2} p(x)dx = 0.3$

   (c) $\int_0^{2.72} p(x)dx = 0.5$

   (d) $\int_0^{\infty} xp(x)dx = 2.65$

   Answer the following questions:

   (a) What is the median annual snowfall (in m)?

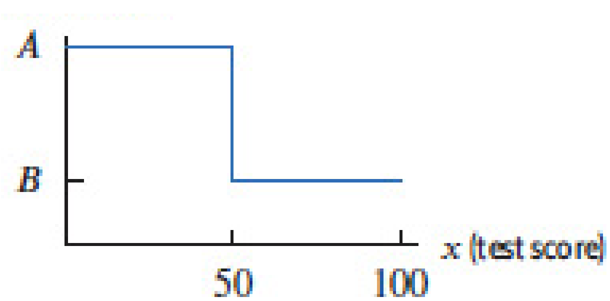
   (b) What is the mean snowfall (in m)?

   (c) What is the probability of annual snowfall between 0.5 and 2 m?

2. A screening test for susceptibility to diabetes reports a numerical score between 0 and 100. A score greater than 50 indicates a potential risk, with some lifestyle training recommended. Results from 200,000 people who were tested show that:

   - 75% received scores evenly distributed between 0 and 50.
   - 25% received scores evenly distributed between 50 and 100.

   The probability density function (pdf) is below.



   (a) Find the values of A and B that make this a probability density function.

   (b) Find the median test score.

   (c) Find the mean test score.

   (d) Give a graph of the cumulative distribution function (cdf) for these test scores.

3. A quantity x has density function $p(x) = 0.5(2 - x)$ for $0 \le x \le 2$ and $p(x) = 0$ otherwise. Find the mean and median of x.

4. A quantity x has cumulative distribution function $P(x) = x - x^2/4$ for $0 \le x \le 2$ and $P(x) = 0$ for $x < 0$ and $P(x) = 1$ for $x > 2$. Find the mean and median of x.

5. While taking a walk along the road where you live, you accidentally drop your glove, but you don't know where. The probability density $p(x)$ for having dropped the glove x kilometers from home (along the road) is
$$p(x) = 2e^{-2x} \text{ for } x \ge 0.$$
   (a) What is the probability that you dropped it within 1 kilometer of home?

   (b) At what distance y from home is the probability that you dropped it within y km of home equal to 0.95?

6. Using Desmos on a screen everyone can see, sketch graphs of the density function of the normal distribution
$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}.$$
   (a) Fix μ (for example, $\mu = 5$) and vary σ (for example $\sigma = 1, 2, 3$).

   (b) Fix σ (for example $\sigma = 1$) and vary μ (for example $\mu = 4, 5, 6$).

   (c) Explain how the graphs confirm that μ is the mean of the distribution, and that σ is a measure of how closely the data is clustered around the mean.

   (d) Using calculus techniques, show that $p(x)$ has a maximum when $x = \mu$. What is that maximum value?

   (e) Show that $p(x)$ has inflection points where $x = \mu + \sigma$ and $x = \mu - \sigma$.