1. Write the following Taylor polynomials:

   (a) the 4th degree T.P. for $f(x) = \frac{1}{1+x}$ near $x = 0$

   (b) the degree 4 T.P for $f(x) = \cos x$ near $x = \pi/2$

   (c) the degree 2 T.P. for $f(x) = \sqrt[3]{1-x}$ near $x = 0$

2. The function $f(x)$ is approximated near $x = 0$ by the third degree Taylor polynomial

$$P_3(x) = 2 - x - x^2/3 + 2x^3.$$

   Give the value of

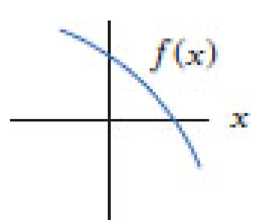
   (a) $f(0)$

   (b) $f'(0)$

   (c) $f''(0)$

   (d) $f'''(0)$

3. Suppose $P_2(x) = a + bx + cx^2$ is the second-degree Taylor polynomial for the function f about $x = 0$. What can you say about the signs of a, b, c if f has the graph given below?



4. The Taylor polynomial of degree 7 of $f(x)$ is given by

$$P_7(x) = 1 - \frac{x}{3} + \frac{5x^2}{7} + 8x^3 - \frac{x^5}{11} + 8x^7.$$

Find the Taylor polynomial of degree 3 of $f(x)$.

5. Use the Taylor approximation for x near 0,

$$\sin x \approx x - \frac{x^3}{3!},$$

to explain why $\lim_{x\to 0} \frac{\sin x}{x} = 1$.

6. The integral $\int_0^1 (\sin t/t)dt$ is difficult to approximate using, for example, left Riemann sums or the trapezoid rule because the integrand $(\sin t)/t$ is not defined at $t = 0$. However, this integral converges; its value is 0.94608... Estimate the integral using Taylor polynomials for $\sin t$ about $t = 0$ of degree 3 and degree 5.

7. For each, use the third degree Taylor polynomial

$$P_3(x) = 4 + 2(x-1) - 2(x-1)^2 + (x-1)^3/2$$

of $f(x)$ about $x = 1$ to find the given value, or explain why you can't.

   (a) $f(1)$
   (b) $f'(1)$
   (c) $f''(0)$
   (d) $f^{(4)}(1)$