Math 156 Applied Honors Calculus II Computer Lab Worksheet Fall 2009

Log in to your umich account, open Maple, then click on **File**, then select **New**, then click on **Worksheet Mode**. Type each command on this page and hit return to get the result. Use the arrow keys to maneuver left/right on a line, or up/down to move to different lines.

Arithmetic

> 2+2;
> 2^6;
> sqrt(64);
> a:=3; *Be sure to include the colon before the equal sign.*
> a^2;
> Pi;
> evalf(Pi); *evalf = evaluate*

You can put several commands on a single line.

> sin(Pi); cos(Pi); tan(Pi);
> erf(0); erf(1); evalf(erf(1)); $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$: *error function (see hw3)*
> abs(-1);
> I; I^2; $\mathrm{I} = \sqrt{-1}$, *an imaginary number, written "i" in math books*
> exp(1); evalf(exp(1));
> exp(Pi*I); *that's right,* $e^{\pi i} = -1$; *you'll learn why in December*

Limits

> with(student); *This loads the student commands.*
> Limit(1/x,x=infinity);
> value(%); *% refers to the expression on the previous line*
> limit(1/x,x=infinity); Limit *displays the limit,* limit *evaluates it.*
> limit(1/x,x=0);
> limit(1/x,x=0,left); limit(1/x,x=0,right); *This gives one-sided limits.*
> limit(x*exp(-x),x=infinity); *Maple knows l'Hopital's rule.*

Plotting

At the top of the screen, click **Maple 12**, then **Preferences**, then **Display**, then in the **Plot display** menu, change **Inline** to **Window**, and click **Apply to Session**. This opens each plot in a separate window.

> plot(1/x,x=0..5,y=0..5); *After viewing, close the plot to avoid clutter.*
> plot([1/x,1/x^2],x=0..5,y=0..5); *Which curve is* $1/x$? ... $1/x^2$?
> plot(tan(x),x=-2*Pi..2*Pi,y=-4..4); $\tan(x)$ *has vertical asymptotes at* $x = \pm\pi/2, \ldots$
> limit(tan(x),x=Pi/2);
> limit(tan(x),x=Pi/2,left); limit(tan(x),x=Pi/2,right);
> plot(arctan(x), x=-2*Pi..2*Pi,y=-4..4); $\arctan(x)$ *has horizontal asymptotes as* $x \to \pm\infty$
> limit(arctan(x),x=infinity);
> plot([exp(-x),exp(-x^2)],x=0..3,y=0..1); *Which curve is* e^{-x}? ... e^{-x^2}?

The next plot is an example of a parametric curve using polar coordinates.

> plot([sin(4*t),t,t=0..2*Pi],coords=polar); *Try changing 4 to 7 (for example).*

Maple can explain each command, e.g. as on the next line.

> ?plot

Riemann Sums

> rightbox(x^2,x=0..1,2); *This plots the right-hand Riemann sum for* $\int_0^1 x^2 dx$ *with* $n = 2$.

> rightsum(x^2,x=0..1,2); evalf(%); *This evaluates the Riemann sum.* $\Delta x = ?$, $x_i = ?$

Repeat these two commands for $n = 4, 8, 16$ using the arrow keys or mouse to position the pointer and change n. Do the results converge to the correct value $\int_0^1 x^2 dx = 1/3 = 0.333\ldots$?

The corresponding commands for the left-hand Riemann sum are **leftbox**, **leftsum**, and for the midpoint rule they are **middlebox**, **middlesum**. Repeat the previous commands, substituting *left* and *middle* in place of *right*. For a given value of n, which type of Riemann sum is the most accurate?

Antiderivatives

> Int(x^n,x); value(%);

> int(x^n,x); Int *displays the integral,* int *evaluates it.*

> int(ln(x),x); diff(%,x);

> int(1/(x^2+1),x); diff(%,x);

> int(1/sqrt(x^2+1),x); diff(%,x); *We'll discuss* $\sinh(x)$ *later in the semester.*

> int(exp(-x^2),x); diff(%,x); *recall* : $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

> f:=x^3/sqrt(1-x^8);

> int(f,x); *We can also use substitution to find the antiderivative, as follows.*

> a:=Int(f,x);

> b:=changevar(x^4=u,a); $u = x^4,\ du = 4x^3 dx$

> c:=value(b);

> d:=subs(u=x^4,c); *This returns to the original variable.*

> diff(d,x); *This checks the answer.*

Definite Integrals

> Int(x,x=a..b); value(%); *Oops - we need to clear variables a and b.*

> a:='a'; b:='b'; Int(x,x=a..b); value(%); ok

> Int(sqrt(1-x^2),x=-1..1); value(%); *area of a semi-circle with radius 1*

> Int(x^(-2),x=1..infinity); value(%);

> Int(x^(-2),x=-1..1); value(%);

> Int(x^(-1),x=1..infinity); value(%);

> Int(exp(-x),x=0..infinity); value(%);

> Int(x*exp(-x),x=0..infinity); value(%); *Maple knows integration by parts.*

> Int(exp(-x^2),x=0..infinity); value(%); *This requires multivariable calculus (Math 255).*

Homework Assignment (hand in with hw4 on Tuesday Oct 6)

In class and on hw2 we computed Riemann sums for the integral $I = \int_0^1 f(x)\,dx$ with $f(x) = e^x$, e^{-x}, and we found that if Δx decreases by a factor of 1/2, then the error in the right-hand Riemann sum R_n decreases by about 1/2 and the error in the midpoint rule M_n decreases by about 1/4. Do the same results hold when $f(x) = \sqrt{x}$? To answer this question, construct a table with the following data (you may use Maple or a calculator). column 1: n (take $n = 2, 4, 8, 16$), column 2: Δx, column 3: R_n, column 4: $|I - R_n|$, column 5: M_n, column 6: $|I - M_n|$. For a given value of n, which method gives a more accurate answer? How do the results for $\sqrt{x}$ compare with the results for e^x, e^{-x}? What is similar? ... different? Explain your observations.