Math 156 Applied Honors Calculus II Fall 2009

hw4 , due: Tuesday, October 6

Remember - explain your reasoning, erase mistakes, write neatly, show intermediate steps.

section 6.4 (work) page 402 / 22 (use $\rho = 1000$ kg/m^3)

(applied project) page 658 / 2 (in part (b), assume the baseball weighs 5 oz; you may use the following conversions: 1 lb = 16 oz, 1 mile = 5280 ft, $g = 32$ ft/sec^2, 1 hr = 3600 sec)

section 8.8 (improper integrals) page 574 / 13 , 22 , 27 , 28 , 52 , 62 , 77

hints: 574 / 13, 22, 27, 28, plot the integrand; 574 / 62, substitute $x = v\sqrt{M/2RT}$

1. In class we used the comparison theorem to show that the improper integral $I = \int_0^\infty e^{-x^2} dx$ is convergent. Now suppose we want to find the numerical value of I. We can't use the FTC because the antiderivative of e^{-x^2} is not an elementary function, but we can approximate the value of I as follows. First write $I = I_1 + I_2$, where $I_1 = \int_0^4 e^{-x^2} dx$ and $I_2 = \int_4^\infty e^{-x^2} dx$.

a) Approximate I_1 using the midpoint rule with $n = 4$ intervals.

b) Show that I_2 satisfies the bound $0 < I_2 < \int_4^\infty e^{-4x} dx < 10^{-7}$. (hint: sketch the functions $y = e^{-x^2}$ and $y = e^{-4x}$ for $x \geq 0$ on the same plot)

c) If we neglect I_2 and take $I \approx I_1$, how large is the error in the approximation? (Use the fact, stated in class, that $I = \int_0^\infty e^{-x^2} dx = \sqrt{\pi}/2 = 0.8862269255$).

2. The Gamma function $\Gamma(x)$ is defined by $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$.

a) Evaluate $\Gamma(1),\ \Gamma(2),\ \Gamma(3)$.

b) Show that $\Gamma(n+1) = n\Gamma(n)$, where n is any non-negative integer.

c) Show that $\Gamma(n+1) = n!$, where $n! = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1$.

note : setting $n = 0$ in (c), we obtain $0! = \Gamma(1)$

3. Find the antiderivative. Derive the answer (don't just use a formula from a table).

a) $\int \sin x \, \cos x \, dx$ b) $\int \sin^2 x \, \cos x \, dx$ c) $\int x\sqrt{x^2 + a^2} \, dx$

announcement

The first midterm exam is on Wednesday, October 14, 6:15-7:45pm in 140 Lorch. If you have a conflict, you must notify your instructor as soon as possible. The exam will cover Appendix E (sigma notation), sections 5.1-5.4 (Riemann sums, area, integrals, FTC), 6.4 (work), 8.8 (improper integrals), 9.1 (arclength), 9.2 (surface area), plus the homework and lecture notes. A review sheet will be distributed before the exam. Calculators are not allowed on the exam. You may use one page (i.e. one side) of notes. We will supply the exam booklets.