Math 156 Applied Honors Calculus II Fall 2009

hw6 , due: Tuesday, October 27

section 8.3 (trig substitution) 531 / 38

section 9.2 (surface area) page 596 / 31 , 33

section 9.3 (center of mass) page 606 / 20 , 22 , 23

chapter 9 (problems plus) page 620 / 4a

1. a) Evaluate the quantity $\left(1+\frac{1}{n}\right)^n$ for $n = 1, 10, 10^2, 10^3, 10^4$ (using a calculator).

b) Show analytically that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$. (Hint: use the fact that $\ln e = 1$.)

2. The purpose of this exercise is to introduce the <u>hyperbolic functions</u> (these are discussed in section 7.6 of the textbook).

a) Define $\cosh x = \frac{e^x+e^{-x}}{2}$ and $\sinh x = \frac{e^x-e^{-x}}{2}$. Show that $\cosh x$ is an <u>even</u> function (i.e. $f(-x) = f(x)$) and $\sinh x$ is an <u>odd</u> function (i.e. $f(-x) = -f(x)$).

b) Sketch the graphs of $\frac{e^x}{2}$, $\frac{e^{-x}}{2}$, $\cosh x$ on the same plot for $-3 \le x \le 3$. Label each curve. Repeat for $\frac{e^x}{2}$, $-\frac{e^{-x}}{2}$, $\sinh x$ (on a new plot).

c) Show that $\cosh^2 x - \sinh^2 x = 1$.

Note: The result in part (c) above says that the point $(X, Y) = (\cosh x, \sinh x)$ lies on the hyperbola $X^2 - Y^2 = 1$ in the XY-plane and it explains why the functions $\cosh x$ and $\sinh x$ are called <u>hyperbolic</u> trigonometric functions. The usual trigonometric functions $\cos x$ and $\sin x$ are sometimes called <u>circular</u> trigonometric functions because they satisfy the equation $\cos^2 x + \sin^2 x = 1$ and so the point $(X, Y) = (\cos x, \sin x)$ lies on the circle $X^2 + Y^2 = 1$. By analogy with the definitions $\cosh x = \frac{e^x+e^{-x}}{2}$ and $\sinh x = \frac{e^x-e^{-x}}{2}$, it can be shown that $\cos x = \frac{e^{ix}+e^{-ix}}{2}$ and $\sin x = \frac{e^{ix}-e^{-ix}}{2i}$, where $i = \sqrt{-1}$. We'll discuss this later in this course.

d) Find $\frac{d}{dx}\cosh x$, $\frac{d}{dx}\sinh x$.

e) Define $\tanh x = \frac{\sinh x}{\cosh x}$. Show that $\tanh x$ is an odd function. Find $\frac{d}{dx}\tanh x$.

f) Evaluate $\lim_{x\to\pm\infty}\tanh x$. Sketch the graph of $\tanh x$ for $-3 \le x \le 3$.