Math 156 Applied Honors Calculus II Fall 2009

hw7 , due: Tuesday, November 3

section 9.3 (center of mass) page 606 / 26 , 33 , 44

hint for 44(c): try $m = n + 1$ for $n = 0, 1, 2, \ldots$

chapter 9 (problems plus) page 621 / 9

hint: you need to derive an equation and find the solution; to find the solution you may use a calculator or the *fsolve* command in Maple (type *?fsolve* in Maple to get the description of this command)

section 9.5 (probability) page 617 / 2 , 3 , 7

1. Evaluate the following limit. $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n$

2. Sketch the region R in the xy-plane and find the center of mass. (Assume $\rho = 1$.)

a) $R = \{(x, y) : 0 \le y \le \sin x, 0 \le x \le \pi\}$ b) $R = \{(x, y) : 0 \le y \le x, 0 \le x \le 1\}$

3. The circle $x^2 + (y - a)^2 = r^2$ is rotated about the x-axis. Assume that $a > r$, so that the resulting shape is a torus. Use the theorem of Pappus to find the volume of the torus.

4. The pdf for a normal distribution is $f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-(x-\mu)^2}{2\sigma^2}}$, where μ , $\sigma > 0$ are constants. Derive the following results. You may use the fact that $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$.

a) $\int_{-\infty}^{\infty} f(x)\, dx \;=\; 1$ b) $\int_{-\infty}^{\infty} x f(x)\, dx \;=\; \mu$ c) $\int_{-\infty}^{\infty} (x - \mu)^2 f(x)\, dx \;=\; \sigma^2$

This verifies that μ is the mean and σ is the standard deviation of the pdf $f(x)$ given above.

5. True or False. Justify your answer.

a) For a normal distribution, the median is equal to the mean.

b) If $(\overline{x}, \overline{y})$ denotes the center of mass of a region in the xy-plane with density $\rho = 1$, then the area of the region to the left of the line $x = \overline{x}$ is the same as the area of the region to the right of the line.

6. Find the antiderivative.

a) $\int \sinh x\, dx$ b) $\int \cosh x\, dx$ c) $\int \tanh x\, dx$ d) $\int \operatorname{sech} x\, dx$, where $\operatorname{sech} x = \frac{1}{\cosh x}$