Math 156 Applied Honors Calculus II Fall 2009

hw9 , due: Tuesday, November 17

section 9.5 (probability) page 618 / 15

section 10.3 (separation of variables) page 643 / 11 , 37 , 39

section 10.4 (exponential growth and decay) page 656 / 8 , 13

section 10.5 (logistic equation) page 666 / 8

section 12.1 (sequences) page 747 / 19 , 32 , 40

section 12.2 (series) page 756 / 11 , 12 , 17 , 41 , 57

1. This exercise refers to sequences. In each case you need to give an example; do this by drawing some points to represent the sequence and giving a formula for the general term a_n.

a) It was stated in class that a convergent sequence is bounded. However, the converse is false, i.e. a bounded sequence is not necessarily convergent. Show this by giving an example of a bounded sequence which is not convergent.

b) It was also stated that if a bounded sequence is either increasing or decreasing, then it does converge. However again the converse is false, i.e. a convergent sequence is not necessarily increasing or decreasing. Show this by giving an example of a convergent sequence which is neither increasing or decreasing.

2. Recall from hw8: $f(x) = f(a) + f'(a)(x-a) + \int_a^x (x-t)f''(t)\,dt$.

a) Now show that $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \int_a^x \frac{(x-t)^2}{2} f'''(t)\,dt$.

(hint: in the result from hw8, set $u = f''(t)$, $dv = (x-t)\,dt$, and integrate by parts)

b) Define the function $T_2(x)$ as below.

$$T_2(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$$

Note that $T_2(x)$ is a quadratic function of x; it is called the Taylor polynomial of degree 2 for $f(x)$ at $x=a$. Show that $T_2(x)$ and $f(x)$ have the same function value, 1st derivative value, and 2nd derivative value at $x = a$. We view $T_2(x)$ as a quadratic approximation to $f(x)$ near the point $x = a$.

c) Note that part (a) says, $f(x) = T_2(x) + \int_a^x \frac{(x-t)^2}{2} f'''(t)\,dt$.

In this case the error is the difference between the given function $f(x)$ and the quadratic approximation $T_2(x)$. Show that the error satisfies the bound $|f(x) - T_2(x)| \le \frac{1}{3!} M_3 |x-a|^3$, where $M_3 = \max|f'''(t)|$. This implies that if x is close to a, then $T_2(x)$ is a very good approximation to $f(x)$.

d) In each case below find $T_2(x)$ and sketch $f(x)$, $T_1(x)$, $T_2(x)$ on the same graph.

(i) $f(x) = e^x,\ a = 0$ (ii) $f(x) = \sin x,\ a = 0$ (iii) $f(x) = \sin x,\ a = \frac{\pi}{4}$

announcement

The 2nd midterm exam is on Wednesday, November 18, 6:15-7:45pm in 140 Lorch Hall. The exam will cover sections 9.3 (center of mass), 9.5 (probability), 10.1 (differential equations), 10.3 (separation of variables), 10.4 (exponential growth and decay), 10.5 (logistic equation), 12.1 (sequences), 12.2 (series), hyperbolic functions, Taylor polynomials. A review sheet will be distributed soon. Calculators are not allowed on the exam. You may use one page (i.e. one side) of notes. We will supply the exam booklets.