Math 156 Applied Honors Calculus II Fall 2009

hw10 , due: Wednesday, December 2

section 10.4 (exponential growth and decay) page 657 / 11

section 12.2 (series) page 756 / 42 , 54 , 68 (note: for #54, sketch by hand)

section 12.3 (integral test for series) page 765 / 5 , 7

section 12.4 (comparison test for series) page 770 / 29 , 37

section 12.5 (alternating series) page 775 / 5

section 12.6 (ratio test) page 782 / 31 , 33

1. Show that the series given below are convergent and in each case find the smallest value of n which ensures that the nth partial sum s_n is accurate to within 10^{-6}.

a) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^4}$ b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^4}$

2. Recall from hw9: $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \int_a^x \frac{(x-t)^2}{2} f'''(t)\,dt$.

a) Now show that $f(x) = f(a)+f'(a)(x-a)+\frac{f''(a)}{2}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\int_a^x \frac{(x-t)^3}{3!} f^{(4)}(t)\,dt$.

(hint: in the result from hw9, set $u = f'''(t)$, $dv = \frac{(x-t)^2}{2}dt$, and integrate by parts)

b) Define the function $T_3(x)$ as below.

$$T_3(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$$

Note that $T_3(x)$ is a cubic function of x; it is called the Taylor polynomial of degree 3 for $f(x)$ at $x=a$. Show that $T_3(x)$ and $f(x)$ have the same function value, 1st derivative value, 2nd derivative value, and 3rd derivative value at $x = a$. We view $T_3(x)$ as a cubic approximation to $f(x)$ near the point $x = a$.

c) Note that part (a) says, $f(x) = T_3(x) + \int_a^x \frac{(x-t)^3}{3!} f^{(4)}(t)\,dt$.

In this case the error is the difference between the given function $f(x)$ and the cubic approximation $T_3(x)$. Show that the error satisfies the bound $|f(x) - T_3(x)| \le \frac{1}{4!}M_4|x-a|^4$, where $M_4 = \max|f^{(4)}(t)|$. This implies that if x is close to a, then $T_3(x)$ is a very very good approximation to $f(x)$.

d) In each case below find $T_3(x)$ and sketch $f(x)$, $T_1(x)$, $T_2(x)$, $T_3(x)$ on the same graph.

(i) $f(x) = e^x,\ a = 0$ (ii) $f(x) = \sin x,\ a = 0$ (iii) $f(x) = \sin x,\ a = \frac{\pi}{4}$