Clarification for lecture 1

We wanted to compute the value of $\sum_{i=1}^{n} i$. Call this sum S. The strategy was to compute the value of $\sum_{i=1}^{n}((i+1)^2 - i^2)$ in two different ways, and equate the two different expressions obtained for this value. The first way to compute it is by the telescoping sum theorem, which says here that

$$\sum_{i=1}^{n}((i+1)^2 - i^2) = (n+1)^2 - 1^2 = n^2 + 2n.$$

The second way to compute it is as follows:

$$\sum_{i=1}^{n}((i+1)^2 - i^2) = \sum_{i=1}^{n}(2i+1) = 2\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1 = 2S + n.$$

Thus

$$n^2 + 2n = \sum_{i=1}^{n}((i+1)^2 - i^2) = 2S + n,$$

and now we solve for S by first subtracting n from both sides to get $n^2 + n = 2S$, and then dividing by 2 to get $(n^2 + n)/2 = S$. We conclude that

$$\sum_{i=1}^{n} i = \frac{n^2 + n}{2}.$$

Let me know if you have any questions!